ORC: An Online Competitive Algorithm for Recommendation and Charging Schedule in Electric Vehicle Charging Network

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ABSTRACT

There is an increasing need for spatial and temporal schedule tailored to the requests and preferences of electric vehicles (EVs) in a network of charging stations. From the perspective of a charging network operator, this paper considers an online decision-making problem that recommends charging stations and the corresponding energy prices to sequential EV arrivals, and schedules the charging allocation to maximize the expected total revenue. To address the uncertainties from future EV arrivals and EVs' choices with respective to recommendations, we propose an Online Recommendation and Charging schedule algorithm (ORC) that is parameterized by a value function for customized designs. Under the competitive analysis framework, we provide a sufficient condition on the value function that can guarantee ORC to be online competitive. Moreover, we design a customized value function based on the sufficient conditions in an asymptotic case, and then rigorously prove the competitive ratio of ORC in the general case. Through extensive experiments, we show that ORC achieves significant increase of revenues compared to benchmark online algorithms.

CCS CONCEPTS

• **Theory of computation** → **Online algorithms**; • **Networks** → **Network economics**.

KEYWORDS

Electric vehicles, online algorithms, recommendation, charging schedule, competitive ratios

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1 INTRODUCTION

Mobility and energy management of a population of electric vehicles (EVs) have drawn great attention from both industry and academy with the increasing penetration of EVs in both public (e.g., EV buses/taxis) and private transportation [10, 13, 20, 25]. For a charging network operator (CNO) who manages multiple publicly accessible charging stations, two types of decisions need to be made upon the arrival of each EV: (i). A *spatial schedule* that decides which station the EV should be assigned/recommended to and (ii). A *temporal schedule* that determines when the EV should be charged at the station decided by the spatial schedule. The following three practical issues arise for a CNO to solve this sequential decision-making problem.

- **Uncertainty**, it is challenging to predict the charging demand (such as energy to be delivered and its deadline) of each EV since it varies with time, location, and the state of each individual EV (e.g., state-of-charge and battery capacity) [1, 19, 24]. A CNO needs to design an *online* algorithm that makes spatial and temporal schedules without knowing the information of future arrivals.
- **Discretion**, unlike managing an EV fleet owned by a private operator, a CNO cannot assign the EVs to charging stations by mandatory orders [10, 13]. One promising solution is to strategically devise a charging offer, which contains a recommended charging station and a corresponding energy price, for each EV to choose by itself. EVs can choose to accept this offer, or reject it and leave for alternative charging facilities.
- **Commitment**, the charging demand is time-varying and geographically unbalanced. As a result, the states of charging stations (e.g., number of idle chargers) at the time when a CNO recommends stations may differ from when an EV arrives at the station for charging. Consequently, the demands of EVs may not be satisfied. To guarantee the quality of service, when making spatial schedules, a CNO needs to make on-arrival commitment [1], by reserving enough charging capacity for future temporal schedules.

In consideration of the three issues above, the key challenge of designing and analyzing an online algorithm lies in making decisions with two types of uncertainties. Type I uncertainty includes the charging demands and contextual information (e.g., location, time, EV brand, etc.) of all future EVs. Type II uncertainty is the randomness that EVs may or may not accept the recommendations provided by a CNO. Note that EVs may choose to reject all recommendations and no revenues can be achieved by both online and offline algorithms. Therefore, instead of focusing on the worst-case scenario for both Type I and II uncertainties, it is desirable to have a less conservative online algorithm that maximizes the expected total revenue averaging over all Type II realizations in the worse-case realization of Type I uncertainty.

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Contributions. In this paper, based on a recommender system formed by a CNO and a network of charging stations, we design an Online algorithm that decides Recommendations and Charging schedules with on-arrival commitment (ORC) for sequential EV arrivals. In more detail, an EV initiates a charging session inquiry, which includes its energy demand and charging deadline, to a CNO through an online platform. Upon receiving the inquiry, an ORC offers that EV a station-price pair, which indicates the energy price if the EV charges at this station. This offer can also be set as an empty entry if no charging station is available. The EV then decides whether to accept it or not. If the recommendation is accepted, the EV will arrive at the recommended station and pay for the charging service with the fixed price in the offer. If the EV rejects the offer, it can either leave for alternative charging facilities, or adjust its charging demand (by reducing its energy demand or extending its departure time) and submit a new inquiry, which may be offered a more satisfactory recommendation. For each accepted offer, an ORC manages to schedule charging that satisfies the EV's demand, i.e., to deliver the required energy by the deadline at the recommended charging station. The objective of an ORC is to maximize the total revenue collected from all EVs.

Value functions (see Definition 3.1) that evaluate the marginal cost for charging an EV at a given charging station are used to parameterize the ORC. As our next result, in Theorem 4.2, we provide a sufficient condition on value functions for ensuring that the ORC has a constant (expected) competitive ratio. Furthermore, based on the sufficient condition, we propose a systematic way for generating an explicit value function with a provable competitive ratio summarized in Theorem 4.4.

To demonstrate the effectiveness of the ORC, we provide a case-study based on geographical data in Hong Kong in Section 6. The (empirical) competitive ratios in our experiments outperform other three online benchmarks.

Related work. A rich literature on online management of EVs has emerged in recent years and the work can be broadly divided into two branches. In the sequel, we overview a collection of the representative results.

Temporal scheduling of EVs. The key idea in temporal scheduling of EVs is treating the EVs as deferrable loads and schedule the charging based on their demands. Along the lines of this idea, previous work has been focusing on coordinating the charging process of a fleet of EVs by temporal load shifting (i.e., temporal schedules) to mitigate their negative impacts on power systems [6, 7] or gain profits [19, 21]. Online algorithms have been considered in [1–3, 9, 9, 19, 23], which aim to achieve a bounded competitive ratio. Some recent work [15, 17, 24] developed auction-based or posted price mechanisms to handle the EVs' choices but the main focus there is temporal scheduling for a single charging station. The authors of [1] reported an impossibility result that there is no competitive online algorithm for on-arrival commitment charging schedule in the general setting. However, this paper will show that our proposed online algorithm ORC can achieve a bounded competitive ratio if more information (e.g., the set of possible prices and the choice probabilities of EVs) is given for decision-making.

Spatial scheduling of EVs. More recent studies [4, 20, 25] indicate that it is also of great importance to recognize EVs as mobile loads, which can be scheduled to be served at different locations (i.e., spatial schedules) to balance the location-dependent EV charging demands. The literature on the spatial scheduling of EVs mainly considers evaluating the system-level benefits of the spatial scheduling. For example, the authors of [4] designed a pricing strategy for routing EVs to charging stations that optimizes the charging loads of different nodes in a transmission network and simultaneously mitigates the congestion of a transportation network. The authors of [25] used a game-theoretic approach to devise a spatial-temporal scheduling for multiple groups of public EVs competing for capacity-limited charging stations. Further investigations on pricing mechanisms for private EVs that charge at public EV charging stations can be found in [13]. Although geographical information is considered, most existing works on spatial scheduling model EV arrivals as a stationary distribution that is known a priori. In contrast to this, realistic EV arrival patterns depend on times and locations and are in general hard to forecast [1, 19, 24]. Designing an online algorithm for spatial scheduling with sequential EV arrivals is therefore still an open and challenging task. There is a theory-to-application gap between existing online algorithms mainly designed for temporal scheduling and practical situations wherein EVs are spatial loads and have their own freedom to make decisions. The main goal of this paper is to close this gap by proposing an online competitive algorithm for joint spatial and temporal scheduling.

2 PROBLEM STATEMENT

This section first presents the online spatial-temporal scheduling problem that an ORC solves. Next, we formally define the expected competitive ratio, as an offline benchmark and then an auxiliary optimization is given whose optimal value bounds the expected competitive ratio from above, based on which the ORC is designed and analyzed.

2.1 Online spatial-temporal scheduling

The CNO manages a set $\mathcal{M} := \{1, \dots, M\}$ of geographically distributed charging stations. Each charging station $m \in \mathcal{M}$ is equipped with b_m chargers, each of which can be used for on/off charging control with a fixed charging rate R_m. Due to varying electricity prices, each station $m \in \mathcal{M}$ is allowed to set their energy prices for charging sessions from a set of *price levels* $\mathcal{R}_m := \{r^{m,j}\}_{j \in \mathcal{J}^m}$, where $\mathcal{J}^m := \{1, \dots, J^m\}$ is the index set of price levels. Without loss of generality, the price levels are sorted in an ascending order, i.e., $0 < r^{m,1} < \cdots < r^{m,J^m}$.

A set $\mathcal{N} := \{1, \dots, N\}$ of EVs sequentially submit charging sessions to the CNO. Consider a time-slotted system[1] indexed by $t \in \mathcal{T} := \{1, \dots, T\}$ with a fixed slot length ΔT. Let a_n, e_n, and d_n denote the submission time, energy demand, and deadline of charging session n, respectively. To model EVs' preferences for stations and prices, we define the following matrix of probabilies.

Choice probabilities. Based on the charging requests and contextual information, we assume each EV has a fixed *choice probability* over the recommended station-price pairs. The choice probability of EV n is denoted by $\boldsymbol{p}_n := \{p_n^{m,j}\}_{m \in \mathcal{M}, j \in \mathcal{J}^m}$ where $p_n^{m,j}$ indicates the preference of an EV n for charging at station m with

[1]Charging sessions can be submitted and processed in real time but the charging schedule has to be done over discrete time steps.

the price $r^{m,j}$. It must satisfy that (i) $0 \le p_n^{m,j} \le 1$, and (ii) it is monotonically non-increasing with respect to price, i.e., $p_n^{m,j} \le p_n^{m,j'}$ for all $j' < j$ and $m \in \mathcal{M}$.

Upon the submission of an EV $n \in \mathcal{N}$, the CNO has access to the following information:

- Charging requirements of an EV n in different charging stations $\{e_{n,m}, \mathcal{T}_{n,m}\}_{m\in\mathcal{M}}$, where $e_{n,m} = \lceil e_n/(R_m \Delta T)\rceil$ is the required number of time slots to charge EV n at station m and $\mathcal{T}_{n,m} := \{a_n + t_{n,m}, \dots, d_n\}$ is the set of feasible time slots for charging EV n at station m, where $t_{n,m}$ denotes the estimated traveling time for EV n to reach station m.
- The matrix of probabilities $\boldsymbol{p}_n$.

The CNO then performs spatial scheduling (i.e, deciding a station-price recommendation) based on the above information and temporal scheduling (i.e, allocating charging time slots) for each charging session. In the following, we introduce more decision variables that the CNO can optimize over to maximize its revenue.

Recommendation decisions. The *recommendation variable* $y_n^{m,j} \in \{0,1\}$ is one if a station-price pair (m, j) is recommended to an EV n and zero otherwise. No recommendation will be made if $y_n^{m,j} = 0$ for all $m \in \mathcal{M}$ and $j \in \mathcal{J}^m$. For the simplicity of presentation and analysis, we assume each EV n only received one recommendation and our proposed scheme can be easily extended to the cases when multiple recommendations are given to each EV.

Charging decisions. Another binary variable $x_{n,t}^{m,j} \in \{0,1\}$ is the *charging variable* that determines whether to charge an EV n at station $m \in \mathcal{M}$ at time $t \in \mathcal{T}_{n,m}$. Again, for the ease of presentation, we restrict the charging levels to be two-states–on and off.

Maximization objective. Given a recommended station-price pair (m, j), if an EV n accepts this offer, it pays $e_n r^{m,j}$ to the CNO and drives to station m for charging. The CNO guarantees to deliver e_n energy to this EV before its deadline d_n at station m. The objective of the CNO is to *maximize the total revenue* collected from all EVs.

After receiving the recommendation pair, the EV n accepts it with probability $p_n^{m,j}$.

Remark 2.1. When choice probabilities are all ones, the spatial decision of the online problem reduces to an assignment problem as EVs will accept any recommendation for sure.

Remark 2.2. (An example of estimating choice probabilities) We can model the preference of EV n over station-price pair (m, j) as

$$f_n(m, j; \boldsymbol{\gamma}_n) = \gamma_n(1) + \gamma_n(2)/t_{n,m} + \gamma_n(3)/(r^{m,j})^2,$$

which is inversely proportional to the driving time to station m and the energy price $r^{m,j}$. $\boldsymbol{\gamma}_n = [\gamma_n(1), \gamma_n(2), \gamma_n(3)]$ customizes the preference of each EV n and can be estimated from historical data by regression. The choice probability of EV n for the station-price pair (m, j) can be estimated by a softmax function:

$$p_n^{m,j} = \frac{\exp\left(f_n(m, j; \boldsymbol{\gamma}_n)\right)}{\sum_{m\in\mathcal{M}} \sum_{j\in\mathcal{J}^m} \exp\left(f_n(m, j; \boldsymbol{\gamma}_n)\right)}. \tag{1}$$

An instance of the online spatial-temporal schedule problem contains three types of information:

- Setup information $\mathcal{S} := \{\mathcal{M}, \{b_m\}_{m\in\mathcal{M}}, \{\mathcal{R}_m\}_{m\in\mathcal{M}}\}$;
- Arrival information $\mathcal{I} := \{\boldsymbol{\theta}_n\}_{n\in\mathcal{N}}$, where the arrival information of EV n is $\boldsymbol{\theta}_n = \{\{e_{n,m}, \mathcal{T}_{n,m}\}_{m\in\mathcal{M}}, \boldsymbol{p}_n\}$;
- Choice information $\mathcal{W} := \{\boldsymbol{w}_n\}_{n\in\mathcal{N}}$, where the choice information of EV n is $\boldsymbol{w}_n := \{w_n^{m,j}\}_{m\in\mathcal{M}, j\in\mathcal{J}^m}$, in which $w_n^{m,j} \in \{0,1\}$ is EV n's choice on the recommendation (m, j) where $w_n^{m,j}$ is a realization of the Bernoulli random variable $W_n^{m,j}$ with success probability $p_n^{m,j}$.

The setup information $\mathcal{S}$ is fixed and known by the CNO a priori while the arrival information and choice information are revealed sequentially, and especially, the choice information depends on previous decisions. Given a setup $\mathcal{S}$, the goal in this paper is to design an online algorithm operated by the CNO that generates station-price recommendations for each EV n and schedules charging based on the arrival information $\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_n$ and the choice information $\boldsymbol{w}_1, \dots, \boldsymbol{w}_{n-1}$. We encode the information available at an online CNO for providing a recommendation for an EV n by

$$\mathcal{F}_n := \{\mathcal{S}, \boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_n, \boldsymbol{w}_1, \dots, \boldsymbol{w}_{n-1}\}. \tag{2}$$

Let $\mathsf{ALG}(\mathcal{I}; \mathcal{W})$ denote the revenue collected from an online algorithm for an arrival instance $\mathcal{I}$. $\mathsf{ALG}(\mathcal{I}; \mathcal{W})$ is a random variable with respective to $\mathcal{W}$, the random choices from EVs . The goal of the online algorithm is to maximize the expected revenue $\mathsf{E}_{\mathcal{W}}[\mathsf{ALG}(\mathcal{I}, \mathcal{W})]$.

Next we define an offline benchmark for the online algorithm. Let $\mathcal{O}_n = \mathcal{F}_n \cup \{\boldsymbol{\theta}_{n+1}, \dots, \boldsymbol{\theta}_N\}$ denote the offline information for decision-making of EV n given the entire arrival information $\mathcal{I}$. In this offline benchmark, even though the choice probabilities of all future EVs are known a priori, the realizations of future EVs' choices, including the current one, are unknown for the offline benchmark. Therefore, the optimal offline algorithm is an adaptive algorithm and the optimal solution is a policy mapping the offline information $\mathcal{O}_n$ to decisions. Let $Y_n^{m,j}(\mathcal{O}_n)$ and $X_{n,t}^{m,j}(\mathcal{O}_n)$ denote the recommendation and charging variables based on $\mathcal{O}_n$. $Y_n^{m,j}(\mathcal{O}_n)$ and $X_{n,t}^{m,j}(\mathcal{O}_n)$ are the outputs of a causal decision maker π whose decisions can only be made based on $\mathcal{O}_n$. The CNO can collect a revenue $e_n r^{m,j}$ from EV n only if the station-price pair (m, j) is offered to EV n (i.e., $Y_n^{m,j}(\mathcal{O}_n) = 1$) and meanwhile the EV accepts this offer (i.e., $W_n^{m,j} = 1$). The total revenue is

$$\mathsf{RVN}(\pi, \mathcal{I}; \mathcal{W}) := \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{j=1}^{J^m} (e_n r^{m,j}) \cdot Y_n^{m,j}(\mathcal{O}_n) \cdot W_n^{m,j},$$

subject to

$$\sum_{t\in\mathcal{T}_{n,m}} X_{n,t}^{m,j}(\mathcal{O}_n) \ge e_{n,m} Y_n^{m,j}(\mathcal{O}_n), \quad \forall n, m, j, \tag{3a}$$

$$\sum_{n=1}^{N} \sum_{j=1}^{J^m} W_n^{m,j} \cdot X_{n,t}^{m,j}(\mathcal{O}_n) \le b_m, \quad \forall m, t, \tag{3b}$$

$$\sum_{m=1}^{M} \sum_{j=1}^{J^m} Y_n^{m,j}(\mathcal{O}_n) \le 1, \quad \forall n, \tag{3c}$$

$$Y_n^{m,j}(\mathcal{O}_n) \in \{0,1\}, \quad \forall n, m, j, \tag{3d}$$

$$X_{n,t}^{m,j}(\mathcal{O}_n) \in \{0,1\}, \quad \forall n, m, j, t \in \mathcal{T}_{n,m}, \tag{3e}$$

where constraint (3a) is the energy constraint that ensures a charging allocation is scheduled to deliver e_n amount of energy at station m if EV n is offered to charge at station m. Constraint (3b) is the

capacity constraint ensuring the total number of EVs that are simultaneously charging at a station cannot exceed the total number of chargers. Constraint (3c) restricts that at most one station-price pair is offered to each EV. We consider the best causal decision maker π^* defined by

$$\pi^* = \arg\max_{\pi} \mathsf{E}_{\mathcal{W}}[\mathsf{RVN}(\pi, \mathcal{I}; \mathcal{W})]. \tag{4}$$

With π^* being used, the optimal revenue for each $\mathcal{W}$ is $\mathsf{OPT}(\mathcal{I}; \mathcal{W})$ and the expectation $\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I}; \mathcal{W})]$ is maximized.

Due to lacking future arrival information, any online algorithm cannot achieve an expected offline revenue $\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I}; \mathcal{W})]$. In this paper, we aim to design an online algorithm that can achieve at least a fraction of the offline revenue $\alpha \mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I}; \mathcal{W})]$, where $0 \le \alpha \le 1$ is a constant and defined as the competitive ratio of the online algorithm. Formally, the competitive ratio is defined as

$$\alpha = \min_{\text{all possible } \mathcal{I}} \frac{\mathsf{E}_{\mathcal{W}}[\mathsf{ALG}(\mathcal{I}; \mathcal{W})]}{\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I}; \mathcal{W})]}. \tag{5}$$

The goal is to design an online algorithm that achieves a competitive ratio as large as possible. Note that α is defined as the ratio of the expected revenues of the online and offline algorithms over EVs' random choices in the worse-case realization of EVs' arrival instances. Different from the conventional competitive analysis that defines the ratio in the worse scenario regarding all uncertainties, our expected competitive ratio is necessary in this problem since EVs may reject all recommendations such that both online and offline algorithms achieve zero revenue, leading to meaningless ratios defined for cases that rarely occur. Before proceeding to our design, for the purpose of competitive analysis, we first introduce an auxiliary optimization, whose optimal value bounds $\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I}; \mathcal{W})]$ from above.

2.2 Auxiliary optimization

The randomness from EVs' choices makes it hard to find the optimal decision maker π^* and compute $\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I}; \mathcal{W})]$. Thus, given the setup $\mathcal{S}$ and arrival information $\mathcal{I}$, we construct an auxiliary optimization which is a deterministic offline optimization that can be used later in our analysis.

$$\overline{\mathsf{OPT}}(\mathcal{I}) = \max_{\boldsymbol{x}, \boldsymbol{y}} \quad \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{j=1}^{J^m} (e_n r^{m,j}) \cdot p_n^{m,j} \cdot y_n^{m,j}, \tag{6a}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}_{n,m}} x_{n,t}^{m,j} \ge e_{n,m} y_n^{m,j}, \quad \forall n, m, j, \tag{6b}$$

$$\sum_{n=1}^{N} \sum_{j=1}^{J^m} p_n^{m,j} x_{n,t}^{m,j} \le b_m, \quad \forall m, t, \tag{6c}$$

$$\sum_{m=1}^{M} \sum_{j=1}^{J^m} y_n^{m,j} \le 1, \quad \forall n, \tag{6d}$$

$$y_n^{m,j} \ge 0, \quad \forall n, m, j, \tag{6e}$$

$$0 \le x_{n,t}^{m,j} \le 1, \quad \forall n, m, j, t \in \mathcal{T}_{n,m}. \tag{6f}$$

Lemma 2.3. *Given a setup $\mathcal{S}$ and an arrival instance $\mathcal{I}$, the optimal objective $\overline{\mathsf{OPT}}(\mathcal{I})$ of the auxiliary optimization* (6) *is an upper bound of the optimal expected revenue $\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I}; \mathcal{W})]$.*

In this auxiliary optimization, all constraints are satisfied in expectation instead of being respected for every sample path in the original problem (3). Thus, this auxiliary problem can be considered as a relaxation of the original problem and hence builds an upper bound. The formal proof of Lemma 2.3 is presented in Appendix A.

3 ORC: AN ONLINE RECOMMENDATION AND CHARGING SCHEDULE ALGORITHM

In this section, we propose an online algorithm, ORC, that decides the recommendation and charging schedule for each EV n based on the casual information $\mathcal{F}_n$. We first introduce a bid-price control policy that is the basis of ORC and then present ORC in detail.

3.1 Basic idea: a bid-price control policy

In the online spatial-temporal schedule problem, a myopic algorithm will recommend the station-price pair that can maximize the instantaneous expected revenue, i.e.,

$$\max_{m \in \mathcal{M}, j \in \mathcal{J}^m} \quad p_n^{m,j} (r^{m,j} e_n). \tag{7}$$

However, the myopic algorithm may not give the desired competitive ratio since it, on the one hand, fails to balance the charging sessions across different stations. As a result, some popular charging stations may get occupied quickly and some future charging sessions that only accept to charge at those stations cannot be served. On the other hand, the myopic algorithm underestimates the EV arrivals in the future (actually assuming no future arrivals). Thus, each station may be occupied by some charging sessions that have arrived earlier but pay less. To deal with these two issues, an online algorithm needs to provide recommendations to balance the charging sessions in different stations and reserve some charging capacity for future arrivals.

Bid-price control is a popular heuristic method in revenue management literature for balancing the inventory of different resources [8, 16] and reserving resources for future usages [12]. The basic idea of bid-price control is to set a threshold (or bid) price for using one unit of each resource. A product is sold only when the offered price by the customer is larger than the sum of threshold prices of the resources that compose this product. The bid-price control is intuitive and easy for implementation. However, its performance highly depends on the threshold prices and there are no general guidelines for designing those prices.

Our proposed ORC adopts the bid-price control policy for making recommendation decisions. In particular, we can consider the number of idle chargers in one time slot at one station as a resource. A charging session can then be considered as a product that needs to consume a bundle of resources to fulfill energy demand. Thus, the core of ORC is to design threshold prices for activating one charger in each time slot at each station based on the casual information $\mathcal{F}_n$ to estimate the marginal charging cost of each charging session at each charging station.

3.2 Algorithm description

A modified bid-price control policy is used for generating the recommendations to vehicles, which gives the online recommendation algorithm shown in Algorithm 1.

Algorithm 1 The proposed ORC(ϕ)

Inputs: Sequential setup information $\mathcal{S}$ and value functions $\phi = \{\phi_m\}_{m\in\mathcal{M}}$.
Initialization: $v^m_{0,t} = 0, \forall t \in \mathcal{T}, m \in \mathcal{M}$.
while a new charging session n arrives **do**
  Observe arrival information $\boldsymbol{\theta}_n = \{\{e_{n,m}, \mathcal{T}_{n,m}\}_{m\in\mathcal{M}}, \boldsymbol{p}_n\}$;
  for each station $m \in \mathcal{M}$ **do**
    Calculate the candidate charging schedule $\boldsymbol{u}^m_n$;
    Set the marginal cost ζ^m_n;
  end for
  Determine the candidate station-price pair (m^*_n, j^*_n);
  if $p^{m^*_n, j^*_n}_n (r^{m^*_n, j^*_n} - \zeta^{m^*_n}_n) e_n \le 0$ **then**
    Offer no recommendations;
  else
    Recommend (m^*_n, j^*_n) to EV n;
    Observe the realization of EV n's choice;
    if EV n accepts the recommendation **then**
      Recompute charging schedule;
      for $t \in \mathcal{T}_{n,m^*_n}$ **do**
        Charge EV n by $x^{m^*_n, j^*_n}_{n,t} = u^{m^*_n}_{n,t}$;
      end for
      Collect revenue $r^{m^*_n, j^*_n} e_n$ from EV n;
      for $m \in \mathcal{M}, t \in \mathcal{T}$, **do**
        Update $v^m_{n,t} = v^m_{n-1,t} + \sum_{j\in\mathcal{J}^m} x^{m,j}_{n,t}$;
      end for
    else
      for $m \in \mathcal{M}, t \in \mathcal{T}$, **do**
        Update $v^m_{n,t} = v^m_{n-1,t}$;
      end for
    end if
  end if
end while

A. Choose a value function. To estimate the marginal cost of a charging session at a particular station, we need to estimate the cost of activating one charger each time at this station. This motivates the following definition of a *value function*. Denote by $w \in [0, 1]$ the *utilization level* that is defined as the ratio of the chargers being used (divided by the total number of chargers).

Definition 3.1 (Value function). A *value function* $\phi_m(w) : [0, 1] \to [0, r^{m,J^m}]$ is a monotonically non-decreasing function that evaluates the cost of activating one additional charger at charger utilization level w for station m. Denote $\phi := \{\phi_m\}_{m\in\mathcal{M}}$.

B. Calculate candidate charging schedule.

The value function defined above can be used for scheduling EV charging. Let $\boldsymbol{u}^m_n := \{u^m_{n,t}\}_{t\in\mathcal{T}_{n,m}}$ (where $u^m_{n,t} \in \{0, 1\}$) denote the candidate charging schedule if EV n charges at station m (i.e., when station m is recommended to EV n and the EV accepts this offer).

We determine $\boldsymbol{u}^m_n$ by solving the cost minimization problem:

$$\min_{\boldsymbol{u}^m_n} \quad \sum_{t\in\mathcal{T}_{n,m}} u^m_{n,t} \phi_m\left(\frac{v^m_{n-1,t}}{b_m}\right), \tag{8a}$$

$$\text{s.t.} \quad \sum_{t\in\mathcal{T}_{n,m}} u^m_{n,t} \ge e_{n,m}, \tag{8b}$$

$$u^m_{n,t} \in \{0, 1\}, \ t \in \mathcal{T}_{n,m}, \tag{8c}$$

where $v^m_{n-1,t} = \sum_{k=1}^{n-1} \sum_{j\in\mathcal{J}^m} x^{m,j}_{k,t}$ is the total number of occupied chargers at time t by the previous $n-1$ EVs at station m. Based on the value function, $\phi_m(v^m_{n-1,t}/b_m)$ is the estimated cost of activating one charger of station m at time t. The dual variable with respect to constraint (8b), ζ^m_n, can be regarded as the marginal cost of charging session n at station m.

C. Set marginal cost.

Let $\bar{\boldsymbol{u}}^m_n$ denote the optimal solution of the problem (8). The marginal cost ζ^m_n of serving the charging session n at station m is

$$\zeta^m_n := \max_{t\in\mathcal{T}_{n,m}: \bar{u}^m_{n,t}=1} \quad \phi_m\left(\frac{v^m_{n-1,t}}{b_m}\right). \tag{9}$$

D. Determine candidate station-price pair.

The threshold price ζ^m_n gives $(r^{m,j} - \zeta^m_n)e_n$, the pseudo-revenue of serving EV n at station m, based on which the CNO determines the recommendation for EV n by solving

$$(m^*_n, j^*_n) = \operatorname*{arg\,max}_{m\in\mathcal{M}, j\in\mathcal{J}^m} \quad p^{m,j}_n (r^{m,j} - \zeta^m_n) e_n. \tag{10}$$

Using ζ^m_n, we can solve the problem (10) and obtain its optimal solution (m^*_n, j^*_n). If the maximum pseudo-revenue is non-positive, i.e., $r^{m^*_n, j^*_n} - \zeta^{m^*_n}_n \le 0$, the CNO offers no recommendation. This bid-price control can achieve our goals of balancing the charging sessions across the stations and reserving the charging capacity for each station. In particularly, given a station m, if the pseudo-revenue is negative for a price level j, the CNO makes no revenue of recommending (m, j) to EV n anymore and the charging capacity at station m will be reserved for future EVs that can accept higher prices. Moreover, according to the expected pseudo-revenue of different stations when serving the same EV, the station with relative less charging sessions is more possibly offered. To sum up, we can set the recommendation variable $\bar{y}^{m,j}_n$ by

$$\bar{y}^{m,j}_n = \begin{cases} 1 & \text{if } m = m^*_n, j = j^*_n, r^{m^*_n, j^*_n} - \zeta^{m^*}_n > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

E. Recompute charging schedule.

The last step is to compute charging schedules $\bar{x}^{m,j}_{n,t}$, once the user accepts the recommendation:

$$\bar{x}^{m,j}_{n,t} = \bar{y}^{m,j}_n \cdot \bar{u}^m_{n,t}, \quad \forall m, j, t \in \mathcal{T}_{n,m}. \tag{12}$$

Note that when the charging resources have been used up by the previous $n-1$ EVs at station m at time t, we have $v^m_{n-1,t} = b_m$. If the optimal solution of problem (8) is $\bar{u}^m_{n,t} = 1$, the marginal cost is $\zeta^m_n = \phi_m(1) = r^{m,J^m}$. Based on the recommendation policy (11), the algorithm will make no recommendations to EV n, which automatically guarantees the feasibility of the solutions.

4 MAIN RESULTS

In this section, we present our main results. The first result is a general statement on sufficient conditions for guaranteeing the competitiveness of our proposed online algorithm ORC. We then design a specific value function for ORC based on this sufficient condition and derive its competitive ratio.

4.1 Sufficient conditions for α-competitiveness

Before proceeding to our first main theorem, we specify our value function (see Definition 3.1) as a class of functions created by concatenating a sequence of functions over the domain $[0, 1]$. Divide the domain $[0, 1]$ into J^m segments $[0, \ell^{m,1}), \ldots, [\ell^{m,J^m-1}, 1]$ with $0 < \ell^{m,1} < \cdots < \ell^{m,J^m-1} < 1$. Formally, we define the following:

Definition 4.1 (Piece-wise value function)**.** A *piece-wise value function* ϕ_m is a value function whose value in each segment ranges between two consecutive prices, i.e.,

$$\phi_m(w) \in [r^{m,j-1}, r^{m,j}), \text{ for all } w \in [\ell^{m,j-1}, \ell^{m,j}).$$

Based on Definition 4.1, the theorem below provides sufficient conditions as a system of ordinary differential equations (ODEs) for ensuring the online algorithm being α-competitive.

Theorem 4.2. *Given a setup $\mathcal{S}$, for any $0 < \alpha \leq 1$, the online algorithm ORC(ϕ) is α-competitive if the following conditions hold:*

(i) There exists a non-decreasing piece-wise value function ϕ_m for all $m \in \mathcal{M}$, which satisfies the following ODEs for all $j \in \mathcal{J}^m$:

$$\begin{cases} \xi^m \phi_m'(w) - \phi_m(w) + r^{m,j} \leq \frac{r^{m,j}}{\alpha}, w \in (\ell^{m,j-1}, \ell^{m,j}), \\ \phi_m(\ell^{m,j-1}) = r^{m,j-1}, \phi_m(\ell^{m,j}) = r^{m,j}, \end{cases} \tag{13}$$

where $\xi^m := (b_m + 1)/b_m$.

(ii) The segment points $\{\ell^{m,j}\}_{j \in \mathcal{J}^m}$ take discrete values, i.e., $\ell^{m,j} \in \{1/b_m, 2/b_m, \ldots, 1\}$, for all $j \in \mathcal{J}^m$

Theorem 4.2 indicates that if we are able to find a value function ϕ that satisfies both (i) and (ii), the competitive ratio of ORC(ϕ) is α. The detailed proof of Theorem 4.2 is deferred to Section 5.1. This sufficient condition is built based on an online primal-dual analysis [5]. Some recent works [11, 18, 22] have also reported similar results that construct a set of ODEs as sufficient conditions for designing online algorithms. Those works derive the ODEs by assuming the capacity to be infinitely-large for taking the limit. However, in this paper, we derive the ODE (13) based on Lagrange's mean value theorem directly without such an assumption. The discrete value constraints are also due to the condition of applying the mean value theorem. Thus, our result is more general and our proof approach may be also useful for existing works to derive sufficient conditions without assuming infinitely-large capacity.

4.2 Value function design

Our next main result gives an explicit construction of a value function satisfying the conditions in Theorem 4.2 asymptotically ($b_m \to \infty$ for all $m \in \mathcal{M}$). In addition, we analyze its corresponding competitive ratio, presented in the following theorem.

Lemma 4.3. *[Asymptotic value function] Given a setup $\mathcal{S}$, as $b_m \to \infty$ for all $m \in \mathcal{M}$, the online algorithm ORC(ϕ) is α_ϕ-competitive if for all $m \in \mathcal{M}$, the value function on the segment $[\hat{\ell}^{m,j-1}, \hat{\ell}^{m,j})$ is*

$$\phi_m(w) = \frac{e^w - e^{\hat{\ell}^{m,j-1}}}{e^{\hat{\ell}^{m,j}} - e^{\hat{\ell}^{m,j-1}}}(r^{m,j} - r^{m,j-1}) + r^{m,j-1}, \tag{14}$$

where segment points $\{\hat{\ell}^{m,j}\}_{j \in \mathcal{J}^m}$ are the solutions of the equations

$$\begin{cases} 1 - e^{-\ell^{m,1}} = \frac{1-e^{-(\ell^{m,2}-\ell^{m,1})}}{1-r^{m,1}/r^{m,2}} = \cdots = \frac{1-e^{-(\ell^{m,J^m}-\ell^{m,J^m-1})}}{1-r^{m,J^m-1}/r^{m,J^m}}, \\ \ell^{m,J^m} = 1, \end{cases}$$

and the competitive ratio is $\alpha_\phi \equiv \min_{m \in \mathcal{M}} 1 - e^{-\hat{\ell}^{m,1}}$.

As $b_m \to \infty$, it follows that $\xi^m \to 1$ and the segment points in the sufficient condition take continuous values. This enables us to derive the value function and its corresponding competitive ratio by binding the inequality in the ODEs (13) and solving it for each segment. The detailed derivation is shown in Section 5.2.1. Based on the asymptotic value function ϕ, our next Theorem shows that if we round the asymptotic value function, parameterized by α_ϕ, a bounded competitive ratio can be guaranteed.

Theorem 4.4. *Given a setup $\mathcal{S}$, the online algorithm ORC($\tilde{\phi}$) is $\alpha_{\tilde{\phi}}$-competitive if for all $m \in \mathcal{M}$, when $q \in \{0, 1/b_m, 2/b_m, \ldots, 1\}$, the value function is, for all $j \in \mathcal{J}^m$,*

$$\tilde{\phi}_m(q) = \begin{cases} \phi_m(\hat{\ell}^{m,j}) = r^{m,j} & q = \frac{\lceil \hat{\ell}^{m,j} b_m \rceil}{b_m}, \\ \phi_m(q) & \frac{\lceil \hat{\ell}^{m,j-1} b_m \rceil}{b_m} < q < \frac{\lceil \hat{\ell}^{m,j} b_m \rceil}{b_m}, \end{cases} \tag{15}$$

where ϕ_m is the asymptotic value function defined in (14), and the competitive ratio is $\alpha_{\tilde{\phi}} \equiv \min_{m \in \mathcal{M}} \frac{1-e^{-\hat{\ell}^{m,1}}}{(b_m+1)(1-e^{-2/b_m})}$.

The value function $\tilde{\phi}$ is a discrete function, which only takes values at discrete points $q \in \{0, 1/b_m, 2/b_m, \ldots, 1\}$. $\tilde{\phi}(q)$ takes the same value as the asymptotic value function $\phi(q)$ except when a segment point of ϕ lies in $(q - 1/b_m, q)$. In that case, $\tilde{\phi}(q)$ is set as the price corresponding to the segment point. Since the charger utilization level only takes discrete values from the same set, $\tilde{\phi}$ can be applied to ORC. Although $\tilde{\phi}$ does not satisfy the ODEs in (13), we are still able to prove that ORC($\tilde{\phi}$) achieves the competitive ratio $\alpha_{\tilde{\phi}}$. Note that $(b_m + 1)(1 - e^{-2/b_m})$ is strictly larger than 1 and goes to 2 as $b_m \to \infty$, the derived competitive ratio for finite segments goes to $\alpha_{\tilde{\phi}} = \alpha_\phi/2$ when b_m goes to infinity. The recovered competitive ratio is smaller than the one proved for the asymptotic case in Theorem 4.3. Despite of this, our results provide a rigorous competitive ratio analysis in case for finite segments. As a comparison, most of the existing work such as [11, 18, 22] only provides results for the asymptotic case. The detailed proof of Theorem 4.4 is presented in Section 5.2.2.

5 COMPETITIVE ANALYSIS

5.1 Proof of Theorem 4.2

Our proof is based on the online primal-dual framework [5]. Its basic idea is to construct a feasible solution to the dual of the upper bound problem (6) based on the decisions made by the online algorithm. Let Dual($\mathcal{I}$) denote the objective value of the dual problem evaluated at the constructed feasible point. Based on weak duality, we have

$$\text{Dual}(\mathcal{I}) \geq \overline{\text{OPT}}(\mathcal{I}) \geq \text{E}_{\mathcal{W}}[\text{OPT}(\mathcal{I}; \mathcal{W})]. \tag{16}$$

Note that $\mathsf{Dual}(\mathcal{I})$ can be regarded as a function of the decisions made by the online algorithms. Thus, if we can show that under the conditions provided in Theorem 4.2, the expected revenue achieved by $\mathsf{ORC}(\phi)$ is bounded by a fraction of the dual objective, i.e.,

$$\mathsf{E}_{\mathcal{W}}[\mathsf{ALG}(\mathcal{I};\mathcal{W})] \geq \alpha \mathsf{Dual}(\mathcal{I}), \tag{17}$$

the online algorithm is α-competitive. Therefore, to prove Theorem 4.2, we will prove two lemmas that show (16) and (17) hold.

The dual of the upper bound problem (6) is

$$\min_{\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\eta},\boldsymbol{\beta}} \quad \sum_{m=1}^{M}\sum_{t=1}^{T}\mu_t^m b_m + \sum_{n=1}^{N}\eta_n + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{j=1}^{J^m}\sum_{t\in\mathcal{T}_{n,m}}\beta_{n,t}^{m,j} \tag{18a}$$

$$\text{s.t.} \quad \eta_n \geq p_n^{m,j}(r^{m,j}-\lambda_n^{m,j})e_n, \forall n,m,j, \tag{18b}$$

$$\beta_{n,m}^{j,t} \geq p_n^{m,j}(\lambda_n^{m,j}-\mu_t^m), \quad \forall n,m,j,t, \tag{18c}$$

$$\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\eta},\boldsymbol{\beta} \geq 0, \tag{18d}$$

where dual variables $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\boldsymbol{\eta}$ correspond to constraints (6b), (6c), and (6d), respectively, and $\boldsymbol{\beta}$ is associated with the constraints $x_{n,t}^{m,j} \leq 1$.

Let $Y_n^{m,j} := Y_n^{m,j}(\mathcal{F}_n)$ and $X_{n,t}^{m,j} := X_{n,t}^{m,j}(\mathcal{F}_n)$ denote the decisions made by the ORC based on casual information $\mathcal{F}_n$. They are random variables with respective to the random choices of the previous $n-1$ customers.

Definition 5.1 (Random dual variables)**.** We construct random dual variables by the decisions $Y_n^{m,j}$ and $X_{n,t}^{m,j}$,

$$M_t^m = \phi_m(V_{N,t}^m/b_m), \quad \forall m,t, \tag{19}$$

$$\Lambda_n^{m,j} = \max_{t\in\mathcal{T}_{n,m}} \phi_m\left(V_{n-1,t}^m/b_m\right)X_{n,t}^{m,j}, \quad \forall n,m,j, \tag{20}$$

$$H_n = \sum_{m=1}^{M}\sum_{j=1}^{J^m} W_n^{m,j}Y_n^{m,j}(r^{m,j}-\Lambda_n^{m,j})e_n, \quad \forall n, \tag{21}$$

$$B_{n,t}^{m,j} = W_n^{m,j}X_{n,t}^{m,j}\left[\Lambda_n^{m,j}-\phi_m\left(V_{n-1,t}^m/b_m\right)\right], \forall n,m,j,t, \tag{22}$$

where $V_{n-1,t}^m := V_{n-1,t}^m(\mathcal{F}_n)$ is the total occupied number of chargers of station m at time t after processing the $(n-1)$-th EV.

Lemma 5.2. *The expected values of the random dual variables*

$$\bar{\mu}_t^m = \mathsf{E}[M_t^m], \bar{\lambda}_n^{m,j} = \mathsf{E}[\Lambda_n^{m,j}], \bar{\eta}_n = \mathsf{E}[H_n], \bar{\beta}_{n,t}^{m,j} = \mathsf{E}[B_{n,t}^{m,j}],$$

are a feasible solution to the dual problem (18).

Lemma 5.2 is proved in Appendix B. Let $\mathsf{Dual}(\mathcal{I})$ denote the dual objective evaluated at the feasible solution $\bar{\mu}_t^m$, $\bar{\lambda}_n^{m,j}$, $\bar{\eta}_n$, and $\bar{\beta}_{n,t}^{m,j}$, we have $\mathsf{Dual}(\mathcal{I}) \geq \overline{\mathsf{OPT}}(\mathcal{I})$ (i.e., Equation (16)) based on the weak duality. We next show Equation (17) holds in Lemma 5.3.

Lemma 5.3. *If the value function ϕ satisfies sufficient conditions in Theorem 4.2, we have* $\mathsf{E}_{\mathcal{W}}[\mathsf{ALG}(\mathcal{I};\mathcal{W})] \geq \alpha\mathsf{Dual}(\mathcal{I})$.

Proof. The expected revenue of the online algorithm is

$$\mathsf{E}_{\mathcal{W}}[\mathsf{ALG}(\mathcal{I};\mathcal{W})] = \mathsf{E}\left[\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{j=1}^{J^m} Y_n^{m,j}W_n^{m,j}e_n r^{m,j}\right],$$

$$= \sum_{n=1}^{N}\mathsf{E}_{\mathcal{F}_n}\left[\sum_{m=1}^{M}\sum_{j=1}^{J^m} Y_n^{m,j}p_n^{m,j}e_n r^{m,j}\right] := \sum_{n=1}^{N}\mathsf{E}_{\mathcal{F}_n}\left[\Delta P_n\right],$$

where ΔP_n is the conditional expected revenue of the n-th EV.

The feasible dual objective value $\mathsf{Dual}(\mathcal{I})$ is

$$\mathsf{E}\left[\sum_{m=1}^{M}\sum_{t=1}^{T}\phi_m\left(\frac{V_{N,t}^m}{b_m}\right)b_m + \sum_{n=1}^{N}H_n + \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{j=1}^{J^m}\sum_{t\in\mathcal{T}_{n,m}}B_{n,t}^{m,j}\right].$$

The first part in the dual objective is

$$\begin{aligned}
&\mathsf{E}\left[\sum_{m=1}^{M}\sum_{t=1}^{T}\phi_m\left(\frac{V_{N,t}^m}{b_m}\right)b_m\right] \\
&\overset{(i)}{=} \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{n=1}^{N}\mathsf{E}\left[\phi_m\left(\frac{V_{n,t}^m}{b_m}\right)-\phi_m\left(\frac{V_{n-1,t}^m}{b_m}\right)\right]b_m, \\
&= \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T}\mathsf{E}\left[\phi_m\left(\frac{V_{n-1,t}^m+X_{n,t}^{m,j}W_n^{m,j}}{b_m}\right)-\phi_m\left(\frac{V_{n-1,t}^m}{b_m}\right)\right]b_m, \\
&= \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T}p_n^{m,j}\mathsf{E}_{\mathcal{F}_n}\left[\phi_m\left(\frac{V_{n-1,t}^m+X_{n,t}^{m,j}}{b_m}\right)-\phi_m\left(\frac{V_{n-1,t}^m}{b_m}\right)\right]b_m.
\end{aligned}$$

The equality (i) holds since $\phi_m(V_{0,t}^m/b_m) = \phi_m(0) = 0$ and the last equality follows by applying tower property. Combining with the conditional expectations $\mathsf{E}[H_n|\mathcal{F}_n]$ and $\mathsf{E}[B_{n,t}^{m,j}|\mathcal{F}_n]$ in Equations (26) and (27), the dual objective can be denoted by

$$\begin{aligned}
\mathsf{Dual}(\mathcal{I}) &:= \sum_{n=1}^{N}\mathsf{E}_{\mathcal{F}_n}\left[\Delta D_n\right], \\
&= \sum_{n=1}^{N}\mathsf{E}_{\mathcal{F}_n}\left[\sum_{m=1}^{M}\sum_{t=1}^{T}p_n^{m,j}\left[\phi_m\left(\frac{V_{n-1,t}^m+X_{n,t}^{m,j}}{b_m}\right)-\phi_m\left(\frac{V_{n-1,t}^m}{b_m}\right)\right]b_m\right. \\
&\quad + \sum_{m=1}^{M}\sum_{j=1}^{J^m}Y_n^{m,j}p_n^{m,j}(r^{m,j}-\Lambda_n^{m,j})e_n \\
&\quad \left. + \sum_{m=1}^{M}\sum_{j=1}^{J^m}\sum_{t\in\mathcal{T}_{n,m}}X_{n,t}^{m,j}p_n^{m,j}\left[\Lambda_n^{m,j}-\phi_m\left(\frac{V_{n-1,t}^m}{b_m}\right)\right]\right],
\end{aligned}$$

where ΔD_n is defined as the conditional expected dual increment by processing the n-th EV. We next prove $\mathsf{E}_{\mathcal{W}}[\mathsf{ALG}(\mathcal{I};\mathcal{W})] \geq \alpha\mathsf{Dual}(\mathcal{I})$ by showing that $\Delta P_n \geq \alpha\Delta D_n, \forall n \in \mathcal{N}$. Note that when no recommendation is offered, i.e., $\sum_{m=1}^{M}\sum_{j=1}^{J^m}Y_n^{m,j} = 0$, $Y_n^{m,j}$ and $X_{n,t}^{m,j}$ are all zeros and $\Delta P_n = \Delta D_n = 0$. Therefore, we only need to consider the case that a station-price pair (m_n^*, j_n^*) is recommended, i.e., $Y_n^{m_n^*,j_n^*} = 1$. Let $\mathcal{T}_n^*$ denote the set of time slots in

which $X_{n,t}^{m_n^*,j_n^*} = 1$. We then have

$$\begin{aligned}
\frac{\Delta D_n}{p_n^{m_n^*,j_n^*}} &= \sum_{t\in\mathcal{T}_n^*}\left[\phi_{m_n^*}\left(\frac{V_{n-1,t}^{m_n^*}+1}{b_{m_n^*}}\right)-\phi_{m_n^*}\left(\frac{V_{n-1,t}^{m_n^*}}{b_{m_n^*}}\right)\right]b_{m_n^*}+\\
&(r^{m_n^*,j_n^*}-\Lambda_n^{m_n^*,j_n^*})e_n+\sum_{t\in\mathcal{T}_n^*}\left[\Lambda_n^{m_n^*,j_n^*}-\phi_{m_n^*}\left(\frac{V_{n-1,t}^{m_n^*}}{b_{m_n^*}}\right)\right],\\
&\overset{(ii)}{=}\sum_{t\in\mathcal{T}_n^*}\left[\phi_{m_n^*}\left(\frac{V_{n-1,t}^{m_n^*}+1}{b_{m_n^*}}\right)-\phi_{m_n^*}\left(\frac{V_{n-1,t}^{m_n^*}}{b_{m_n^*}}\right)\right](b_{m_n^*}+1)\\
&\quad+r^{m_n^*,j_n^*}\cdot e_n-\sum_{t\in\mathcal{T}_n^*}\phi_{m_n^*}\left(\frac{V_{n-1,t}^{m_n^*}+1}{b_{m_n^*}}\right),\\
&\overset{(iii)}{=}\sum_{t\in\mathcal{T}_n^*}\left[\frac{b_{m_n^*}+1}{b_{m_n^*}}\phi'_{m_n^*}\left(w_t^{m_n^*}\right)+r^{m_n^*,j_n^*}-\phi_{m_n^*}\left(\frac{V_{n-1,t}^{m_n^*}+1}{b_{m_n^*}}\right)\right],\\
&\overset{(iv)}{\le}\sum_{t\in\mathcal{T}_n^*}\left[\xi^{m_n^*}\phi'_{m_n^*}\left(w_t^{m_n^*}\right)+r^{m_n^*,j_n^*}-\phi_{m_n^*}\left(w_t^{m_n^*}\right)\right],\\
&\overset{(v)}{\le}\sum_{t\in\mathcal{T}_n^*}\left[r^{m_n^*,j_n^*}-r^{m_n^*,j_t}+\frac{r^{m_n^*,j_t}}{\alpha}\right],\\
&\overset{(vi)}{\le}\frac{r^{m_n^*,j_n^*}}{\alpha}\cdot e_n=\frac{1}{\alpha}\frac{\Delta P_n}{p_n^{m_n^*,j_n^*}}.
\end{aligned}$$

By substituting $\sum_{t\in\mathcal{T}_n^*}\Lambda_n^{m_n^*,j_n^*}=e_n\Lambda_n^{m_n^*,j_n^*}$ and arranging the terms, we can easily check the equality (ii). Based on Lagrange's mean value theorem, there exists $w_t^{m_n^*}\in[V_{n-1,t}^{m_n^*}/b_{m_n^*},(V_{n-1,t}^{m_n^*}+1)/b_{m_n^*}]$ such that $\forall t\in\mathcal{T}_n^*$,

$$\phi_{m_n^*}\left((V_{n-1,t}^{m_n^*}+1)/b_{m_n^*}\right)-\phi_{m_n^*}\left(V_{n-1,t}^{m_n^*}/b_{m_n^*}\right)=\phi'_{m_n^*}\left(w_t^{m_n^*}\right)\frac{1}{b_{m_n^*}}.$$

We can then get equality (iii). Since $(V_{n-1,t}^{m_n^*}+1)/b_{m_n^*}\ge w_t^{m_n^*}$ and the value function is non-decreasing, the inequality (iv) holds. Let $t_n^*=\max_{t\in\mathcal{T}_n^*}w_t^{m_n^*}$. Then $w_{t_n^*}^{m_n^*}$ is the largest charger utilization level, at which one more charger is scheduled to be activated (i.e., $X_{n,t_n^*}^{m_n^*,j_n^*}=1$). Based on Equation (13), if $w_t^{m_n^*}\in[\ell^{m_n^*,j_t-1},\ell^{m_n^*,j_t})$, we have inequality (v). Note that $r^{m_n^*,j_t}$ increases with the increase of $w_t^{m_n^*}$. We then have $r^{m_n^*,j_t}\le r^{m_n^*,j_{t_n^*}}$. The value of $r^{m_n^*,j_{t_n^*}}$ is determined by which segment $w_{t_n^*}^{m_n^*}$ lies in. Recall the online algorithm will recommend (m_n^*,j_n^*) only if $r^{m_n^*,j_n^*}>\zeta_n^{m_n^*}=\phi_{m_n^*}(V_{n-1,t_n^*}^{m_n^*}/b_{m_n^*})$. Thus, we have $V_{n-1,t_n^*}^{m_n^*}/b_{m_n^*}\in[\ell^{m_n^*,j_n^*-1},\ell^{m_n^*,j_n^*})$. Since the segment points only take discrete values, $w_{t_n^*}^{m_n^*}\le(V_{n-1,t_n^*}^{m_n^*}+1)/b_{m_n^*}\le\ell^{m_n^*,j_n^*}$. Thus, we have $r^{m_n^*,j_t}\le r^{m_n^*,j_{t_n^*}}=r^{m_n^*,j_n^*}$ and the last inequality (vi) holds. This completes the proof. □

5.2 Proof for Theorem 4.4

To design the value function provided by Theorem 4.4, we first prove Lemma 4.3 and show how to design an asymptotic value function when $b_m\to\infty$ based on the sufficient conditions in Theorem 4.2. When b_m is a finite value, we design a discrete value function by rounding the asymptotic value function, and rigorously prove the competitive ratio it can achieve.

5.2.1 Asymptotic Case ($b_m\to\infty$). Without the constraints of the discreteness as $b_m\to\infty,\forall m\in\mathcal{M}$, the value function can be derived by changing inequality in (13) to equality and solving the ODEs with boundary conditions for each segment.

The general solution of the j-th segment is $\phi_m(w)=a^{m,j}e^w-(1/\alpha^{m,j}-1)r^{m,j}$, where $a^{m,j}$ and $\alpha^{m,j}$ are parameters to be determined. Substituting the boundary conditions we have

$$\begin{cases}a^{m,j}e^{\ell^{m,j-1}}-(1/\alpha^{m,j}-1)r^{m,j}=r^{m,j-1},\\ a^{m,j}e^{\ell^{m,j}}-(1/\alpha^{m,j}-1)r^{m,j}=r^{m,j},\end{cases}$$

and the parameters can be solved as

$$a^{m,j}=\frac{r^{m,j}-r^{m,j-1}}{e^{\ell^{m,j}}-e^{\ell^{m,j-1}}},\quad \alpha^{m,j}=\frac{1-e^{-(\ell^{m,j}-\ell^{m,j-1})}}{1-r^{m,j-1}/r^{m,j}}.$$

Note that the competitive ratio $\alpha=\min_{m\in\mathcal{M},j\in\mathcal{J}^m}\alpha^{m,j}$ is determined by the minimum $\alpha^{m,j}$ over all stations and all segments. Thus, in order to maximize the minimum $\alpha^{m,j}$, for each station m, we set $\alpha^{m,j}$ of each segment to be equal, namely,

$$\alpha^m=1-e^{-\ell^{m,1}}=\cdots=\frac{1-e^{-(\ell^{m,J^m}-\ell^{m,J^m-1})}}{1-r^{m,J^m-1}/r^{m,J^m}}. \tag{23}$$

Lemma 5.4. *For $\forall m\in\mathcal{M}$, there exists a solution $\{\hat{\ell}^{m,j}\}_{j\in\mathcal{J}^m}$ to Equation (23) and $\ell^{m,J^m}=1$.*

The proof of Lemma 5.4 and the computation method for $\hat{\ell}^{m,j}$ have been presented in Appendix C. Based on Lemma 5.4, the competitive ratio is determined by $\alpha=\min_{m\in\mathcal{M}}\alpha^m=\min_{m\in\mathcal{M}}1-e^{-\hat{\ell}^{m,1}}$. By substituting α^m and $\{a^{m,j}\}_{j\in\mathcal{J}^m}$, the asymptotic value function (14) can be derived.

5.2.2 General Case (Finite b_m). If the number of chargers is finite, we design the value function $\tilde{\phi}$ by rounding the asymptotic value function ϕ as shown in Equation (15). Recall $\{\hat{\ell}^{m,j}\}_{j\in\mathcal{J}^m}$ are the segment points of the asymptotic value function ϕ and unnecessarily take discrete values. Thus, we cannot expect $\mathrm{ORC}(\tilde{\phi})$ can achieve a competitive ratio as good as that of the asymptotic case. We next prove the competitive ratio of $\mathrm{ORC}(\tilde{\phi})$.

To analyze $\mathrm{ORC}(\tilde{\phi})$, we can follow the proof of Theorem 4.2 until the procedure to bound the ratio of the primal and dual increments ΔP_n and ΔD_n. The arguments in the proof of Lemma 5.3 fail when $q<\hat{\ell}^{m,j}<q+1/b_m$, where $q=V_{n-1,t}^m/b_m$ is the discrete charger utilization level. This is because in this case, ϕ_m is not differentiable in the interval $(q,q+1/b_m)$ at the point $\hat{\ell}^{m,j}$ and hence the Lagrange's mean value theorem cannot be applied to achieve equality (iii). To bound the ratio of primal and dual increments, we prove the inequality (24) in Lemma 5.5.

Lemma 5.5. *The value function $\tilde{\phi}$ satisfies $\forall m\in\mathcal{M},\forall j\in\mathcal{J}^m$, when $\lceil\hat{\ell}^{m,j-1}b_m\rceil/b_m\le q<\lceil\hat{\ell}^{m,j}b_m\rceil/b_m$, $q\in\{0,1/b_m,\ldots,1\}$,*

$$\left[\tilde{\phi}_m(q+1/b_m)-\tilde{\phi}_m(q)\right]b_m-\tilde{\phi}_m(q)+r^{m,j}\le\frac{r^{m,j}}{\alpha}(b_m+1)(1-e^{-2/b_m}). \tag{24}$$

The proof of Lemma 5.5 is presented in Appendix D. Inequality (24) is the discrete version of the ODE (13). Based on this inequality, we can bound the ratios of primal and dual increments as follows.

$$\frac{\Delta D_n}{p_n^{m_n^*,j_n^*}} = \sum_{t\in\mathcal{T}_n^*}\left\{\left[\phi_{m_n^*}\left((V_{n-1,t}^{m_n^*}+1)/b_{m_n^*}\right)-\phi_{m_n^*}\left(V_{n-1,t}^{m_n^*}/b_{m_n^*}\right)\right]b_{m_n^*}\right.$$
$$\left.+r^{m_n^*,j_n^*}-\phi_{m_n^*}\left((V_{n-1,t}^{m_n^*}+1)/b_{m_n^*}\right)\right\},$$
$$\leq \sum_{t\in\mathcal{T}_n^*}\left[r^{m_n^*,j_n^*}+\left[\frac{(b_m+1)(1-e^{-2/b_m})}{\alpha}-1\right]r^{m_n^*,j_t}\right],$$
$$\leq \frac{(b_m+1)(1-e^{-2/b_m})}{\alpha}\frac{\Delta P_n}{p_n^{m_n^*,j_n^*}}.$$

Thus, the ORC($\tilde{\phi}$) can achieve a competitive ratio of $\alpha/[(b_m+1)(1-e^{-2/b_m})]$.

6 EXPERIMENTAL RESULTS

In this section, we provide experimental results for our proposed ORC algorithm. We construct the value function ϕ in the asymptotic case based on the setup information and then derive the rounding version $\tilde{\phi}$ based on Equation (15). This case study aims to evaluate the empirical performance of ORC($\tilde{\phi}$) and compare it with other benchmark online algorithms.

6.1 Simulation parameters

6.1.1 **Real-world transportation network.** We focus on a simplified transportation network in Hong Kong. The city is divided into 18 districts, which are connected by 32 main roads as shown in Figure 1. The traveling time through each main road is estimated based on Google map data. We assume that there are four public charging stations that can provide charging services for private EVs. The setup information of the four stations is listed in Table 1.

6.1.2 **Generating EV requests.** Each arriving session $n \in \mathcal{N}$ in the experiments is a tuple of random variables $(A_n, E_n, D_n) \in \mathbb{R}_+^3$ where A_n is the submission time; E_n is the energy to be delivered and D_n is its deadline.

The arrival times $\{A_n : n \in \mathcal{N}\}$ are generated according to a non-homogeneous Poisson process, with arrival rate being the product of the time-varying demand function depicted in Figure 2(a) and the average total number of EV arrivals, which is 600 multiplied by a *load factor* $\rho > 0$. We generate EV requests with varying $\rho \in \{0.6, \ldots, 1.6\}$ to investigate the impact of the total number of requests on the performance of online algorithms. The demand function describes the percentage of the hourly refueling demand of gasoline stations within one week [14]. Therefore, the total number of arriving sessions N is a Poisson random variable with mean 600ρ. The total number of arriving sessions are further divided into 18 districts proportionally according to its corresponding normalized population density, which is visualized in Figure 2(b).

After assigning an arriving session $n \in \mathcal{N}$ to the districts, we estimate the driving times $t_{n,m}$ to the four stations based on the shortest paths in the transportation network shown in Figure 1. The energy demand E_n is a uniform random variable defined on $[12, 24]$ (kWh). We set the deadline as $D_n = A_n + t_{n,m} + M_n + U$ where $A_n + t_{n,m}$ is when the EV (if it accepts the recommendation) arrives at the charging station m; $M_n := E_n/R_n$ is its minimum charging time and U is a random variable uniformly distributed in $[0, 2]$ (hours). The charging rate is set as $R_n = 6, \forall n \in \mathcal{N}$ (kW).

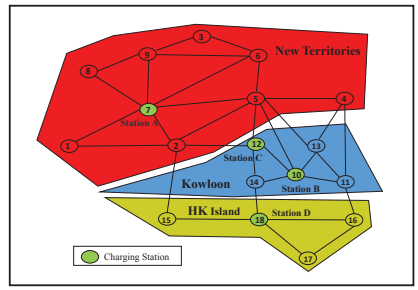
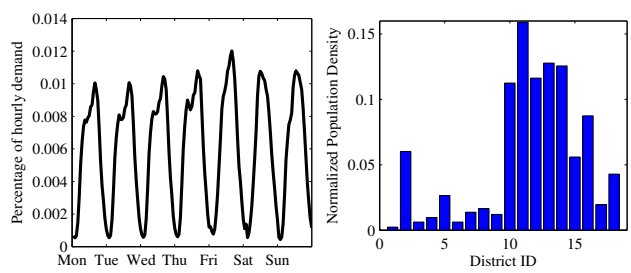


Figure 1: Simplified transportation network of Hong Kong.

Table 1: Setup of Charging Stations

Station ID	Location	No. of Chargers	Prices (cents/kWh)
A	District 7	10	$\{40, 55, 75\}$
B	District 10	10	$\{50, 70, 90\}$
C	District 12	20	$\{50, 70, 90\}$
D	District 18	20	$\{60, 90, 110\}$



(a) Demand for EV charging (b) Population density of 18 districts

Figure 2: Time-varying demand for EV charging and geographically distributed population density.

6.1.3 **Estimating choice probabilities.** We use the choice model described in Remark 2.2 to estimate the choice probability. Particularly, we sample the three preference parameters $\boldsymbol{\gamma}_n$ uniformly from $[0, 1]$, $[20, 30]$, and $[12000, 14000]$ for each EV n.

6.2 Benchmarks

We compare our ORC($\tilde{\phi}$) with the following three benchmark online algorithms. These benchmarks use the same scheduling algorithm for EV charging by greedily assigning the energy to the time slots with lowest utilization of chargers. They, however, differ in the sense that they adopt different rules for selecting the recommendations.

- Myopic. This algorithm recommends (m, j) that maximizes the expected revenue of the current EV by solving the problem (7) without considering future EV arrivals and the imbalance of the charging demand over different stations.

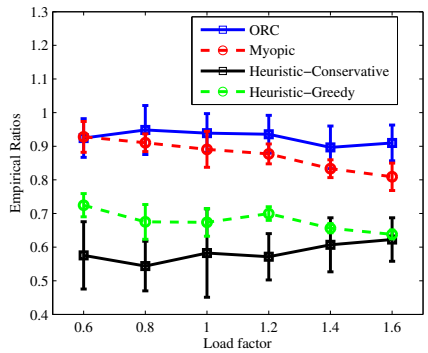


Figure 3: Comparison of empirical ratios achieved by different online algorithms. Results are averaged over 100 independent simulations.

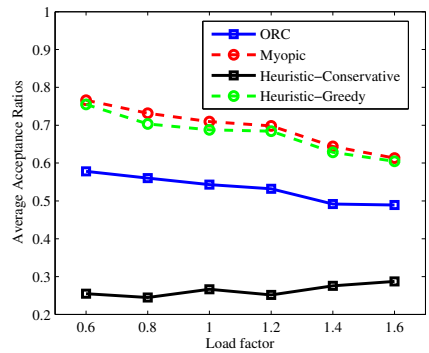


Figure 4: Comparison of the average acceptance ratios achieved by different online algorithms. Results are averaged over 100 independent simulations.

- Heuristic-Greedy. This algorithm recommends the nearest charging station that is feasible to schedule the charging with the lowest price.
- Heuristic-Conservative. In contrast to Heuristic-Greedy, this algorithm offers the nearest station at the highest price.

We compute the total revenue collected by each online algorithm listed above. In each independent simulation, the arriving sessions $\{(A_n, E_n, D_n) : n \in \mathcal{N}\}$ are generated randomly as described in Section 6.1.2 and the EV accepts the recommendation according to the choice probabilities in (1). The final results are averaged over 100 independent simulations.

Let R_{ALG} denote the total revenue of an online algorithm ALG in each simulation and let $R_{\overline{\mathsf{OPT}}}$ denote the optimal value of the upper bound problem (6), which is linear and can be solved by the interior-point algorithm. We compare $\mathsf{ORC}(\tilde{\phi})$ with the three benchmark algorithms based on their empirical ratio, defined as

$$\text{Empirical Ratio of ALG} = R_{\mathsf{ALG}}/R_{\overline{\mathsf{OPT}}}. \tag{25}$$

Since it is computationally difficult to obtain the optimal offline decision π^* in (4), we choose the optimal benchmark of the empirical ratio as the upper bound of the expected optimal revenue. Therefore, the average of the empirical ratio is a lower bound of the competitive ratio (5). Moreover, the empirical ratio is defined for each realization of EVs' random choices, and hence, the total collected revenue sometimes can be greater than the upper bound of the expected revenue. Thus, the empirical ratio may be larger than one for certain realizations. But the average empirical ratio corresponding to each arrival instance (i.e., average of 100 simulations) is guaranteed to be smaller than one.

6.3 Performance evaluation

The numerical results are shown in Figures 3 and 4, with load factors selected from $\{0.6, \ldots, 1.6\}$.

Figure 3 compares the empirical ratios that are achieved by our proposed ORC and other three benchmark online algorithms. For each arrival instance, the empirical ratios of ORC generally outperform those of all benchmark algorithms and ORC achieves the largest average empirical ratio for all arrival instances. The figure also highlights that the average empirical ratio for ORC is very close to 1, which is much larger than the theoretical competitive ratio 0.254 for this simulation setup. This theoretical value is calculated by solving Equation (28) to obtain $\{\hat{\ell}^{m,1}\}_{\forall m \in \mathcal{M}}$ based on bisection search, and then substituting it to the expression of $\alpha_{\tilde{\phi}}$ in Theorem 4.4. The main reason is that the competitive ratio is derived for the worse-case scenario for all possible arriving instances. Moreover, with the increase of the load factor ρ, the performance gap between ORC and Myopic increases. This is because balancing the demand over stations and reserving capacity for future arrivals become more important in high-demand cases, which is also the reason for that the greedy and conservative heuristics perform worse and better, respectively as ρ becomes larger.

Figure 4 displays the average acceptance ratios, i.e., the ratio of the number of served charging sessions and that of the total charging sessions. Myopic and Heuristic-Greedy serve more charging sessions in all cases since both algorithms recommend available charging stations and allocate available chargers in an aggressive manner without reserving some chargers for future EV arrivals, which may accept higher prices. In comparison, our proposed ORC achieves a medium average acceptance ratio to trade off the aggressiveness and conservativeness when allocating the resources without future information. In this way, ORC achieves a much better expected total revenue as shown in Figure 3.

7 CONCLUSIONS

In this paper we introduce an online algorithm, ORC, that can jointly decide recommendation and charging schedules for sequential EV arrivals in a network of charging stations. Sufficient conditions for ensuring that ORC has a constant competitive ratio are provided. Based on the conditions, we design an explicit value function for ORC and its competitive ratio is also given. Numerical experiments validate the effectiveness of ORC by showing that it outperforms other benchmark online algorithms.

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A PROOF OF LEMMA 2.3

Let $\{\hat{Y}_n^{m,j}(\mathcal{O}_n), \hat{X}_{n,t}^{m,j}(\mathcal{O}_n)\}$ denote the decisions made by the optimal decision rule π^* for EV n with the offline information $\mathcal{O}_n$. $\hat{Y}_n^{m,j}(\mathcal{O}_n)$ and $\hat{X}_{n,t}^{m,j}(\mathcal{O}_n)$ only depend on the realizations of EVs' choices, i.e., $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{n-1}$. Therefore, $\hat{Y}_n^{m,j}(\mathcal{O}_n)$ and $\hat{X}_{n,t}^{m,j}(\mathcal{O}_n)$ are both independent of $W_n^{m,j}$. Let $\bar{y}_n^{m,j} = \mathsf{E}[\hat{Y}_n^{m,j}(\mathcal{O}_n)]$ and $\bar{x}_{nt}^{m,j} = \mathsf{E}[\hat{X}_{n,t}^{m}(\mathcal{O}_n)]$, where the expectation is taken with respective to the previous $n-1$ EVs' random choices. The optimal expected revenue can be denoted by

$$\begin{aligned}\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I};\mathcal{W})] &= \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{j=1}^{J}(e_n r^{m,j}) \cdot \mathsf{E}_{\mathcal{W}}[W_n^{m,j} \cdot \hat{Y}_n^{m,j}(\mathcal{O}_n)],\\ &= \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{j=1}^{J}(e_n r^{m,j}) \cdot p_n^{m,j} \cdot \bar{y}_n^{m,j},\end{aligned}$$

which is the objective of the problem (6) when $y_n^{m,j} = \bar{y}_n^{m,j}$. If we can show $\{\bar{y}_n^{m,j}\}_{n,m,j}$ and $\{\bar{x}_{n,t}^{m,j}\}_{n,t,m,j}$ are a feasible solution to the problem (6), it follows $\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I};\mathcal{W})] \le \overline{\mathsf{OPT}}(\mathcal{I})$. Since constraints (3a)-(3e) are respected by the optimal decision rules for any realizations of $\mathcal{W}$, we can take expectation on both sides of these constraints and this leads to the constraints (6b)-(6f) with $\{\bar{y}_n^{m,j}\}_{n,m,j}$ and $\{\bar{x}_{n,t}^{m,j}\}_{n,t,m,j}$ being substituted. Note that we use the independence of $\hat{X}_{n,t}^{m,j}(\mathcal{O}_n)$ and $W_n^{m,j}$ when taking expectation for (3b). Thus, $\{\bar{y}_n^{m,j}\}_{n,m,j}$ and $\{\bar{x}_{n,t}^{m,j}\}_{n,t,m,j}$ are a feasible solution to the problem (6) and hence $\mathsf{E}_{\mathcal{W}}[\mathsf{OPT}(\mathcal{I};\mathcal{W})]$ is an upper bound of $\overline{\mathsf{OPT}}(\mathcal{I})$.

B PROOF OF LEMMA 5.2

Conditional on $\mathcal{F}_n$ or equivalently the random choices of the previous $n-1$ EVs $\{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{n-1}\}$, $Y_n^{m,j}$, $X_{n,t}^{m,j}$ and $V_{n-1,t}^{m}$ are all deterministic values. We then have the conditional expectation

$$\begin{aligned}\mathsf{E}[H_n|\mathcal{F}_n] &= \mathsf{E}\left[\sum_{m=1}^{M}\sum_{j=1}^{J^m} Y_n^{m,j} W_n^{m,j}(r^{m,j} - \Lambda_n^{m,j})e_n|\mathcal{F}_n\right]\\ &= \sum_{m=1}^{M}\sum_{j=1}^{J^m} Y_n^{m,j} p_n^{m,j}(r^{m,j} - \Lambda_n^{m,j})e_n, \qquad (26)\\ &\overset{(i)}{=} \max\left\{\max_{m\in\mathcal{M}, j\in\mathcal{J}^m} p_n^{m,j}(r^{m,j} - \Lambda_n^{m,j})e_n, 0\right\},\\ &\ge p_n^{m,j}(r^{m,j} - \Lambda_n^{m,j})e_n, \forall m \in \mathcal{M}, j \in \mathcal{J}^m.\end{aligned}$$

Equality (i) holds since $Y_n^{m,j}$ is determined based on Equations (10) and (11) to choose the maximum non-negative $p_n^{m,j}(r^{m,j} - \Lambda_n^{m,j})$. By taking expectation on both sides of above equation with respective to $\mathcal{F}_n$ and applying tower property, we show that $\bar{\eta}_n$ and $\bar{\lambda}_n^{m,j}$ satisfy the constraint (18b), namely,

$$\begin{aligned}\bar{\eta}_n &= \mathsf{E}[H_n] = \mathsf{E}_{\mathcal{F}_n}[\mathsf{E}[H_n|\mathcal{F}_n]]\\ &\ge \mathsf{E}_{\mathcal{F}_n}[p_n^{m,j}(r^{m,j} - \Lambda_n^{m,j})e_n]\\ &= p_n^{m,j}(r^{m,j} - \bar{\lambda}_n^{m,j})e_n, \forall m \in \mathcal{M}, j \in \mathcal{J}^m.\end{aligned}$$

Next, by taking conditional expectation on $B_{n,t}^{m,j}$, we have

$$\begin{aligned}\mathsf{E}[B_{n,t}^{m,j}|\mathcal{F}_n] &= X_{n,t}^{m,j} p_n^{m,j}[\Lambda_n^{m,j} - \phi_m(V_{n-1,t}^m/b_m)], \\ &\overset{(ii)}{\geq} p_n^{m,j}[\Lambda_n^{m,j} - \phi_m(V_{n-1,t}^m/b_m)], \\ &\geq p_n^{m,j}[\Lambda_n^{m,j} - \phi_m(V_{N,t}^m/b_m)].\end{aligned} \tag{27}$$

When $X_{n,t}^{m,j} = 1$, inequality (ii) holds naturally. When $X_{n,t}^{m,j} = 0$, there exist two cases. If the station-price (m, j) is not offered, i.e., $Y_n^{m,j} = 0$, $\Lambda_n^{m,j} = 0$ by definition in (20) and inequality (ii) holds. If (m, j) is offered while the EV n is not scheduled to charge at time t, we have $\Lambda_n^{m,j} - \phi_m(V_{n-1,t}^m/b_m) \leq 0$, i.e., the cost of activating one more charger at time t is no less than the marginal cost of the charging session n at station m. Thus, inequality (ii) still holds in this case. Since $V_{n-1,t}^m \leq V_{N,t}^m$ and the value function ϕ_m is non-decreasing, the last inequality holds. Finally, we take expectation with respective to $\mathcal{F}_n$ on both sides of above inequality and we show $\bar{\mu}_t^m$, $\bar{\lambda}_n^{m,j}$, and $\bar{\beta}_{n,t}^{m,j}$ satisfy the constraint (18c).

$$\begin{aligned}\bar{\beta}_{n,t}^{m,j} &= \mathsf{E}[B_{n,t}^{m,j}] = \mathsf{E}_{\mathcal{F}_n}[\mathsf{E}[B_{n,t}^{m,j}|\mathcal{F}_n]] \\ &\geq \mathsf{E}_{\mathcal{F}_n}[p_n^{m,j}[\Lambda_n^{m,j} - \phi_m(V_{N,t}^m/b_m)]], \\ &= p_n^{m,j}(\bar{\lambda}_n^{m,j} - \bar{\mu}_t^m), \forall n \in \mathcal{N}, m \in \mathcal{M}, j \in \mathcal{J}^m, t \in \mathcal{T}_{n,m}.\end{aligned}$$

C PROOF OF LEMMA 5.4

Define the length of the j-th segment as $\sigma^{m,j} = \ell^{m,j} - \ell^{m,j-1}, \forall j \in \mathcal{J}^m$. Based on Equation (23), $\sigma^{m,j}$ can be represented as a function of $\sigma^{m,1}$, i.e.,

$$\sigma^{m,j} = -\ln\left[\frac{r^{m,j-1}}{r^{m,j}} + \left(1 - \frac{r^{m,j-1}}{r^{m,j}}\right)e^{-\sigma^{m,1}}\right], j = 2, \ldots, J^m.$$

Since $\sum_{j\in\mathcal{J}^m} \sigma^{m,j} = 1$, we have

$$e^{-\sigma^{m,1}} \prod_{j=2}^{J^m}\left[\frac{r^{m,j-1}}{r^{m,j}} + \left(1 - \frac{r^{m,j-1}}{r^{m,j}}\right)e^{-\sigma^{m,1}}\right] = e^{-1}. \tag{28}$$

Define $g(\sigma^{m,1})$ as the left hand side of the above equation. $g(\sigma^{m,1})$ is a non-increasing function in $[0, 1]$ with $g(0) = 1$ and

$$g(1) = e^{-1}\prod_{j=2}^{J^m}\left[\frac{r^{m,j-1}}{r^{m,j}}(1 - e^{-1}) + e^{-1}\right] < e^{-1}.$$

Thus, there must exist a solution to $g(\sigma^{m,1}) = e^{-1}$ and hence there exists segments points $\{\hat{\ell}^{m,j}\}_{j\in\mathcal{J}^m}$ that satisfy Equation (23). Since $g(\sigma^{m,1})$ is monotonic in $[0, 1]$, bisection search can be used to find $\sigma^{m,1}$. We can then compute all segment points $\{\hat{\ell}^{m,j}\}_{j\in\mathcal{J}^m}$.

D PROOF OF LEMMA 5.5

We check the inequality in the following three cases.

Case (i): when $q = \lceil \hat{\ell}^{m,j} b_m \rceil / b_m - 1/b_m$, we have

$$\begin{aligned}&\left[\tilde{\phi}_m(q + 1/b_m) - \tilde{\phi}_m(q)\right] b_m + r^{m,j} - \tilde{\phi}_m(q) \\ &\overset{(i)}{=} \left[\phi_m(\hat{\ell}^{m,j}) - \phi_m(q)\right] b_m + \phi_m(\hat{\ell}^{m,j}) - \phi_m(q), \\ &\overset{(ii)}{=} \frac{r^{m,j} - r^{m,j-1}}{e^{\hat{\ell}^{m,j}} - e^{\hat{\ell}^{m,j-1}}}\left[e^{\hat{\ell}^{m,j}} - e^q\right](b_m + 1), \\ &= r^{m,j} \cdot \frac{1 - r^{m,j-1}/r^{m,j}}{1 - e^{-(\hat{\ell}^{m,j} - \hat{\ell}^{m,j-1})}}\left[1 - e^{q - \hat{\ell}^{m,j}}\right](b_m + 1), \\ &\overset{(iii)}{=} \frac{r^{m,j}}{\alpha}\left[1 - e^{q - \hat{\ell}^{m,j}}\right](b_m + 1) \leq \frac{r^{m,j}}{\alpha}(b_m + 1)(1 - e^{-1/b_m}),\end{aligned}$$

where equalities (i) and (ii) are obtained by substituting $\tilde{\phi}_m$ and ϕ_m in Equations (14) and (15), and equality (iii) is from Equation (23).

Case (ii): when $\lceil \hat{\ell}^{m,j-1} b_m \rceil / b_m < q < \lceil \hat{\ell}^{m,j} b_m \rceil / b_m - 1/b_m$.

$$\begin{aligned}&\left[\tilde{\phi}_m(q + 1/b_m) - \tilde{\phi}_m(q)\right] b_m + r^{m,j} - \tilde{\phi}_m(q) \\ &= [\phi_m(q + 1/b_m) - \phi_m(q)] b_m + \phi_m(\hat{\ell}^{m,j}) - \phi_m(q), \\ &= \frac{r^{m,j} - r^{m,j-1}}{e^{\hat{\ell}^{m,j}} - e^{\hat{\ell}^{m,j-1}}}\left[(b_m - (b_m + 1)e^{-1/b_m})e^{q+1/b_m} + e^{\hat{\ell}^{m,j}}\right], \\ &\overset{(iv)}{\leq} \frac{r^{m,j} - r^{m,j-1}}{e^{\hat{\ell}^{m,j}} - e^{\hat{\ell}^{m,j-1}}}\left[(b_m - (b_m + 1)e^{-1/b_m})e^{\hat{\ell}^{m,j}} + e^{\hat{\ell}^{m,j}}\right], \\ &= \frac{r^{m,j}}{\alpha}(b_m + 1)(1 - e^{-1/b_m}),\end{aligned}$$

where inequality (iv) holds since $b_m - (b_m + 1)e^{-1/b_m} \geq 0$ for $b_m = 1, 2, \ldots$, and $q + 1/b_m \leq \hat{\ell}^{m,j}$.

Case (iii): when $q = \lceil \hat{\ell}^{m,j-1} b_m \rceil / b_m$.

$$\begin{aligned}&\left[\tilde{\phi}_m(q + 1/b_m) - \tilde{\phi}_m(q)\right] b_m + r^{m,j} - \tilde{\phi}_m(q) \\ &= \left[\phi_m(q + 1/b_m) - \phi_m(\hat{\ell}^{m,j-1})\right] b_m + \phi_m(\hat{\ell}^{m,j}) - \phi_m(\hat{\ell}^{m,j-1}), \\ &= \frac{r^{m,j} - r^{m,j-1}}{e^{\hat{\ell}^{m,j}} - e^{\hat{\ell}^{m,j-1}}}\left[(b_m - (b_m + 1)e^{\hat{\ell}^{m,j-1} - q - 1/b_m})e^{q+1/b_m} + e^{\hat{\ell}^{m,j}}\right], \\ &\overset{(v)}{\leq} \frac{r^{m,j} - r^{m,j-1}}{e^{\hat{\ell}^{m,j}} - e^{\hat{\ell}^{m,j-1}}}\left[(b_m - (b_m + 1)e^{\hat{\ell}^{m,j-1} - q - 1/b_m})e^{\hat{\ell}^{m,j}} + e^{\hat{\ell}^{m,j}}\right], \\ &\leq \frac{r^{m,j}}{\alpha}(b_m + 1)(1 - e^{-2/b_m}).\end{aligned}$$

Note that $-2/b_m \leq \hat{\ell}^{m,j-1} - q - 1/b_m \leq -1/b_m$. Thus, $b_m - (b_m + 1)e^{\hat{\ell}^{m,j-1} - q - 1/b_m} \geq b_m - (b_m + 1)e^{-1/b_m} \geq 0$ and inequality (v) holds. Combining the three cases, we have the inequality (24).