Robust Provisioning of Demand-Side Flexibility Under Electricity Price Uncertainty

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***Abstract*—With the successful integration of renewable energy sources to the grid, the volatility of supply in the system will increase and therefore a gap between power generation and demand on the grid may occur, which causes the grid frequency to move away from its nominal value. To avoid this scenario, frequency regulation service has been introduced and demand-side flexible loads have been widely considered in recent years. In this paper, we focus on the flexibility of an HVAC system in the building sector and propose a robust optimization approach to improve the user's decision making subject to the electricity price uncertainty. In particular, by explicitly taking into account the price uncertainty in the demand-side frequency regulation, we develop an intelligent building energy management system. The deterministic and robust solutions are compared to explain the influence of price uncertainty on the users' contribution in the frequency regulation service and daily energy payment. The importance of comfort weight factor on the demand-side power consumption profile as well as the corresponding up and down reserve are also investigated. According to the numerical analysis, the robust optimization approach may bring about monetary savings in the electricity bill of the user, particularly in a drastic uncertainty level.**

I. Introduction

The growing number and increasing energy requirements of electrical devices in residential and commercial buildings are pushing up peak demands on utility networks. One way to meet the higher peak demands is to build extra power plants, which requires a significant investment by utility companies. The other approach is integration of renewable and distributed energy resources, which can, however, present challenges due to the intermittency of wind, solar PV, run of river hydro and tidal. Specifically, current climate prediction techniques are not capable of providing a proper estimation of power production variations, and therefore huge unpredicted power generation might happen, which must be handled in an optimal way as the power balance needs to be maintained at all times. To deal with such problems, demand response can be performed by designing incentives to induce lower electricity usage when system reliability is jeopardised or to increase consumption when generation from renewable sources is high [1]-[2].

Frequency Regulation (FR) is a type of Ancillary Services (AS) that imposes short-term changes in electricity usage to maintain the desired electrical frequency for the grid to function normally. Devices such as hot water heaters, thermal systems and plug-in hybrid electric vehicles have some inherent flexibilities to act as demand-side regulating resources and they can respond to the PJM frequency regulation signal [3]-[6]. The participating home users receive monetary compensation for their modified consumption pattern. However, such a modification causes inconvenience, since it deviates from the preferable or customary usage patterns. The objective of each end user is to determine a consumption profile, based on the day-ahead reserve rewards and electricity price, that can strike an optimal balance of this trade-off by minimizing a cost function. This trade-off can be captured through a weight factor that is proportional to the inconvenience caused [7]. A priori reserve scheduling allows buildings to participate in markets for reserve without compromising occupant comfort. This issue was addressed in [8] by developing an MPC-based method to quantify the flexibility of a commercial building and a contractual framework to declare it to the utility. In [9], the authors proposed a robust control framework incorporating energy-constrained frequency signals for reliable scheduling and the provision of frequency reserve by aggregations of commercial buildings.

In this work, we will investigate the influence of electricity price uncertainty on users' decision making and solve a robust optimization problem for a given capacity reward and uncertain electricity price. The aim is to minimize the total cost, defined as the sum of electricity payment and discomfort cost. Improving the customer experience of contribution to the ancillary service with better customers' decisions drives engagement and improves satisfaction. The electricity prices of the day-ahead market may experience an uncertainty due to reasons such as: competition between price makers, contingencies of transmission network and generating units, weather, volatile renewable energy sources and demand uncertainty [10]. The difference between the day-ahead price and real time price represents the loss or a gain in monetary terms.

Robust optimization has been applied to the uncertain electricity price by several authors [11]-[12]. In this paper, we apply the same framework and characterize the uncertain price by the so-called uncertainty set, in which the uncertain price can take values. Initially, we utilize a typical uncertainty set for the price to build a computationally tractable uncertainty set for the electricity price and compare the deterministic and robust solutions. Thereafter, we consider the historical data of the electricity price to choose an appropriate hourly uncertainty set for the robust problem using empirical statistics. According to the analysis of the historic data provided in [13], the average price is not similar in different time slots. For example, relative deviations of hourly prices from their corresponding mean values are almost negligible during the

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morning and they may increase significantly during some hours in the afternoon. In general, the most important time is during hot summer afternoon. During the fall, winter, and spring, the residential real time pricing typically remains low, but can spike due to extreme weather conditions. Thus, it is assumed that the total time horizon is split into multiple intervals, in which the average price uncertainty is a fixed value (e.g. almost zero during the morning and night hours and a constant value in the afternoon, estimated according to the historical data). For those time intervals with a very small price uncertainty, the solution is equivalent to the deterministic one, while during the other periods of time, the solution is robust in the corresponding uncertainty set. In this way, the degree of conservatism is adjusted according to the historical data.

The organization of the paper is as follows. In section II, we describe the system model and problem formulation, and transform a bi-level optimization problem into a single level optimization structure. In section III, a particular focus is placed on describing the sensitivity analysis of the system to price and comfort setting. In section IV, we present numerical results and a performance analysis of the system obtained from simulations. Section V concludes this paper.

II. System Model and Problem Formulation

A. Assumptions

1) We focus on the day-ahead market and assume the day is divided into 24 equal periods. The corresponding set of time slots is denoted by $T = \{1, 2, \cdots, 24\}$. Flexibility from the demand-side is offered one day-ahead and the residential or commercial sectors can contribute to system flexibility through load increase or curtailment of the building thermal system.

2) According to the contract, the utility company charges the customer for its reference power consumption, u_t, based on the real time price irrespective of the deviation from the reference power consumption, which might be imposed by the regulation signal the next day. The user gains rewards appropriate to the declared reserve.

3) We adopt the temperature evolution dynamic equation:

$$x_{t+1} = (1-\alpha)x_t - \beta u_t + \alpha a_t, \tag{1}$$

where x_t and a_t are the indoor and outdoor temperatures at hour t, respectively. Parameters α and β are defined in terms of building insulation parameters and the efficiency of thermal unit, respectively. The system control input is the thermal unit power consumption u_t.

B. Objective Function and Constraints

The optimal decision should be taken at each stage to not only reduce the total payment but also to capture the dissatisfaction caused due to deviation from the reference consumption. Thus, the optimization problem includes two parts, the total energy payment considering the electricity price and reward as the first term, and a dissatisfaction function as the second term, which is indicated by the quadratic form of the deviation from the desired comfort level, as formulated in problem (2a). Hourly price announced to the residential sectors a day-ahead is denoted by $\tilde{p}_t$. We assume knowledge on uncertainty is captured in an uncertainty set $\mathbf{P}$, which is bounded in the interval $[\tilde{p}_t - p_t^-, \tilde{p}_t + p_t^+]$, where p_t^+ and p_t^- are the maximum up and down deviations from the mean electricity price, respectively. Different rewards can be offered by the utility company for the increase/decrease of users' energy consumptions under the supply surplus/deficit, which are defined by r_t^+ and r_t^-, respectively. We call $\delta \geq 0$ the comfort satisfaction weight factor. Each building has a nominal power consumption, u_t, based on the residents' comfort requirements. The users may offer some positive or negative deviations from their reference consumption, which are given by υ_t^+ (up reserve) and υ_t^- (down reserve), respectively. Based on the day-ahead offered reserve, the utility company may change the control input within the lower envelope $(u_t - \upsilon_t^-)$ and upper envelope $(u_t + \upsilon_t^+)$, which are bound within u_{min} and u_{max}. Constraints (2b-2d) are the state space modelings of the building thermal system, based on equation (1), where $A = (1-\alpha)$, $B = -\beta$ and $w_t = \alpha a_t$. They demonstrate how the current state x_t, control input u_t, and weather condition w_t affect the state in the future time step. The maximum allowed deviation of states from the desired level $\hat{x}_t$ is specified in constraints (2e-2g) by the notation y_{max}. Weather prediction uncertainty is not taken into account in this paper.

$$\min_{u_t, \upsilon_t^+, \upsilon_t^-} \quad \sum_{t \in T} \left[\mathcal{C}_1(p_t, u_t) + \mathcal{C}_2(x_t, x_t^+, x_t^-) - \mathcal{R}(\upsilon_t^-, \upsilon_t^+, r_t^-, r_t^+)\right] \tag{2a}$$

$$\begin{aligned}
\textbf{subject to} \quad & x_{t+1} = Ax_t + Bu_t + w_t && \text{(2b)}\\
& x_{t+1}^+ = Ax_t + B(u_t + \upsilon_t^+) + w_t && \text{(2c)}\\
& x_{t+1}^- = Ax_t + B(u_t - \upsilon_t^-) + w_t && \text{(2d)}\\
& |x_t - \hat{x}_t| \leq y_{max} && \text{(2e)}\\
& |x_t^+ - \hat{x}_t| \leq y_{max} && \text{(2f)}\\
& |x_t^- - \hat{x}_t| \leq y_{max} && \text{(2g)}\\
& u_{min} \leq u_t \leq u_{max} && \text{(2h)}\\
& u_{min} \leq u_t + \upsilon_t^+ \leq u_{max} \quad , \quad \upsilon_t^+ \geq 0 && \text{(2i)}\\
& u_{min} \leq u_t - \upsilon_t^- \leq u_{max} \quad , \quad \upsilon_t^- \geq 0 && \text{(2j)}\\
& p_t \in \mathbf{P} && \text{(2k)}
\end{aligned}$$

where the cost of power consumption, the cost of discomfort, and the reserve reward are, respectively,

$$\mathcal{C}_1(p_t, u_t) = p_t u_t,$$
$$\mathcal{C}_2(x_t, x_t^-, x_t^+) = \delta[(x_t - \hat{x}_t)^2 + (x_t^+ - \hat{x}_t)^2 + (x_t^- - \hat{x}_t)^2],$$
$$\mathcal{R}(\upsilon_t^-, \upsilon_t^+, r_t^-, r_t^+) = r_t^+ \upsilon_t^+ + r_t^- \upsilon_t^-.$$

C. Robust Optimization Formulation

Robust optimization is an approach to deal with the uncertainty in optimization problems especially when the probability distribution function (PDF) of the uncertainty cannot be easily described. Here we consider a similar technique to model the uncertainty associated with the electricity price, the traditional one-range robust optimization approach that was developed in [14]. Since the actual concern of the user is the electricity price being more than the forecasted value, we study the case that the actual price is more than the prediction. Uncertainty modeling of the price can be expressed as

$$p_t = \tilde{p}_t + p_t^+ \epsilon_t, \quad 0 \le \epsilon_t \le 1, \tag{3}$$

For simplicity and a compact representation of the objective, we define $\boldsymbol{x_t} = (x_t, x_t^-, x_t^+)$, $\boldsymbol{v_t} = (v_t^-, v_t^+)$, $\boldsymbol{r_t} = (r_t^-, r_t^+)$, and $\mathcal{J}(\boldsymbol{x_t}, \boldsymbol{v_t}, \boldsymbol{r_t}) = \mathcal{C}_2(x_t, x_t^-, x_t^+) - \mathcal{R}(v_t^-, v_t^+, r_t^-, r_t^+)$. The last term will be used in the objective problem for the rest of the paper. Inserting equation (3) into the optimization problem (2a), the new problem is expressed as

$$\min_{u_t, \boldsymbol{v_t}} \quad \sum_{t \in T} \left[\tilde{p}_t u_t + p_t^+ \epsilon_t u_t + \mathcal{J}(\boldsymbol{x_t}, \boldsymbol{v_t}, \boldsymbol{r_t}) \right] \tag{4a}$$

$$\textbf{subject to} \quad \sum_{t \in T} \epsilon_t \le \Gamma \quad , \quad 0 \le \epsilon_t \le 1 \tag{4b}$$

$$(2b) - (2j).$$

Γ is the uncertain coefficient that can be tolerated and it allows the decision-maker to control the degree of conservatism of the solution. We are considering the uncertainty hidden in the day-ahead market. Thus, notice that $0 \le \Gamma \le 24$ must hold for equation (4b) to be meaningful. Under no prediction error (i.e., $\Gamma = 0$), the problem reduces to its deterministic counterpart. In contrast, $\Gamma = 24$ means 100% prediction error and each uncertain parameter can take its worst case value. Taking a value of Γ between 0 and 24 allows the decision-maker to achieve a trade-off between the nominal performance of the deterministic model and the risk protection of the most conservative model. We will investigate the user's decision behavior under different values of Γ in the next section. The robust counterpart of problem (4a) is represented by [15]:

$$\min_{u_t, \boldsymbol{v_t}} \sum_{t \in T} \left[\tilde{p}_t u_t + \mathcal{J}(\boldsymbol{x_t}, \boldsymbol{v_t}, \boldsymbol{r_t}) \right] + \left\{ \begin{array}{l} \max\limits_{\epsilon_t} \sum\limits_{t \in T} p_t^+ \epsilon_t u_t \\ \textbf{subject to} \\ (4b) \end{array} \right\} \tag{5}$$

$$\textbf{subject to} \quad (2b) - (2j).$$

The above bi-level problem consists of an outer optimization associated with an inner optimization. The objective function of the inner optimization is $p_t^+ \epsilon_t u_t$, maximized over ϵ_t, subject to the constraint $(4b)$. The outer minimization controls three decision variables, u_t, v_t^+, v_t^-, to reduce the undesired impacts of the electricity price uncertainty. Using the Lagrangian duality associated with the inner maximization, problem (5) is equivalent to (see Appendix for the proof)

$$\min_{u_t, \boldsymbol{v_t}, \zeta_t, \beta} \quad \sum_{t \in T} \left[\tilde{p}_t u_t + \mathcal{J}(\boldsymbol{x_t}, \boldsymbol{v_t}, \boldsymbol{r_t}) \right] + \beta \Gamma \tag{6a}$$

$$\textbf{subject to} \quad \beta + \zeta_t \ge p_t^+ u_t \quad , \quad \beta, \zeta_t \ge 0 \tag{6b}$$

$$(2b) - (2j).$$

We refer to ζ_t and β as the Lagrange multipliers associated with the inequality constraint (4b).

III. Sensitivity analysis of system flexibility

In this part, we elaborate the sensitivity of the proposed system from two aspects. First, the ratio of electricity price and reserve rewards, and second, the user's comfort setting.

A. Sensitivity analysis to price

The amount of reserve under uncertain price depends on several factors, such as the ratio of the electricity price p_t and reserve payments r_t^- and r_t^+ at each hour, as well as the comfort satisfaction weight factor δ. Comparing the coefficients of u_t in the deterministic ($\Gamma = 0$) and robust problems ($\Gamma \neq 0$), the hourly relationship between electricity price and reserve rewards can be divided in to three cases as follows.

$$Case - 1a : \begin{cases} \tilde{p}_t - r_t^- + r_t^+ \le 0 \\ \tilde{p}_t + p_t^+ \epsilon_t - r_t^- + r_t^+ > 0 \end{cases} \tag{7}$$

$$Case - 1b : \begin{cases} \tilde{p}_t - r_t^- + r_t^+ \le 0 \\ \tilde{p}_t + p_t^+ \epsilon_t - r_t^- + r_t^+ < 0 \end{cases} \tag{8}$$

$$Case - 2 : \begin{cases} \tilde{p}_t - r_t^- + r_t^+ \ge 0 \\ \tilde{p}_t + p_t^+ \epsilon_t - r_t^- + r_t^+ > 0 \end{cases} \tag{9}$$

In $Case - 1b$, the electricity price is very cheap compared with the reserve rewards, even under the price uncertainty. Thus, the user prefers to have more power consumption and to offer more down reserve. However, due to the expensive electricity price in $Case - 2$, the user offers more up reserve. $Case - 1a$ shows a special case, where the day-ahead electricity price is low, but the real time price may dominate the reserve rewards, depending on the hourly uncertainty level $p_t^+ \epsilon_t$. Therefore, an uncertain price may have an influence on the user's decision under this case, which is the main focus of the paper.

B. Sensitivity analysis to comfort setting

In this part, we consider a system, in which the building occupants can switch between three different comfort satisfaction levels, including strict, mild and loose. On the strict comfort satisfaction adjustment, the comfort weight factor δ has the highest value compared with the mild and loose. It is clear that the user may offer a very limited flexibility to the utility company with the strict comfort satisfaction due to the high expectation of the comfort level. However, the proportion of frequency regulation contribution is enhanced in the mild and loose comfort settings. Figure 1 is a very simple representation of the comparison between frequency regulation contribution and comfort level in different adjustments of comfort satisfaction, but the accurate proportion depends on the value of δ. The energy payment is larger with the strict comfort setting since the user cannot gain sufficient revenue by offering up and down reserve. Hence, a deterministic contract is defined, in which the user can declare the system flexibility for different time slots within a day and correspondingly gets rewarded based on the reserve provided to the utility company. Further investigation of the proposed system's sensitivity analysis is performed in the following sections.

IV. Performance analysis

In this section, profound sensitivity analysis is provided in terms of volatility in day-ahead market prices. We solve the deterministic and robust optimization problems and compare

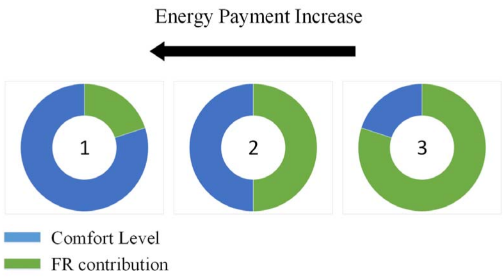

Fig. 1. Comparison of users' frequency regulation contribution and comfort satisfaction in three different comfort settings: 1-Strict, 2-Mild, 3-Loose.

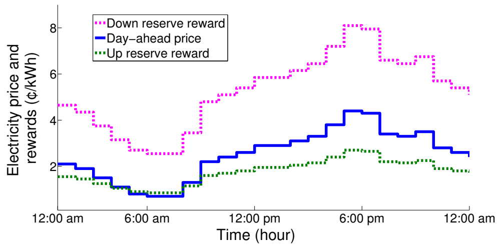

Fig. 2. Day-ahead electricity price and hourly rewards for up/down reserve.

the results. For a more comprehensive illustration of the system, a simulation is initially carried out, given a fixed uncertainty set across one day in summer. Afterwards, we rely on the historical data to construct an appropriate hourly uncertainty set and investigate the simulation results across a week to investigate its long term effect. For the simulation, we considered a room with a cooling system. We assume the temperature reference level to be $21°C$ and the maximum allowed violation to be $2°C$. The day-ahead electricity price is according to PJM day-ahead hourly market price [13]. The algorithm is implemented in MATLAB and is solved by CVX.

A. One day analysis with constant uncertainty set

Hourly up and down rewards and the day-ahead price used for the simulations are shown in Figure 2. The maximum allowed deviation of electricity price from the predicted value is assumed to be $2¢/kWh$. The simulation was carried out for different values of Γ to investigate the resident's behavior against the different levels of price uncertainty. The results in Figure 3 indicate that, with higher values of Γ coefficient, the users tend to offer less down reserve but higher up reserve. Figure 4 demonstrates the user's decision regarding the reference power consumption, and up and down reserve when there is no uncertainty in the electricity price ($\Gamma = 0$). A similar analysis was carried out for a maximum uncertain coefficient of $\Gamma = 24$, which is shown in the same figure as the green dashed line. Correspondingly, the indoor temperature profiles under the two values of Γ are illustrated in Figure 4. Thus, given $\Gamma = 0$, the reference energy profile is very close to the upper envelope and with the added uncertainty in the electricity price, it moves towards the lower envelope. Two cases are studied, as follows.

1) Case I: This case is for a comparison of deterministic and robust solutions under the worst realization of price uncertainty. Daily energy payment is calculated for deterministic and robust solutions under different values of Γ and the results

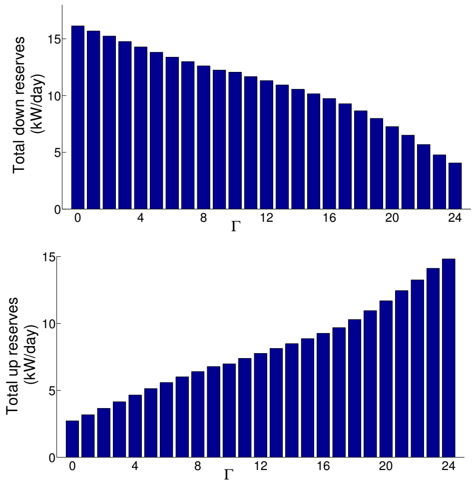

Fig. 3. Total up/down daily reserve for different Γ, given $\delta = 0.1$.

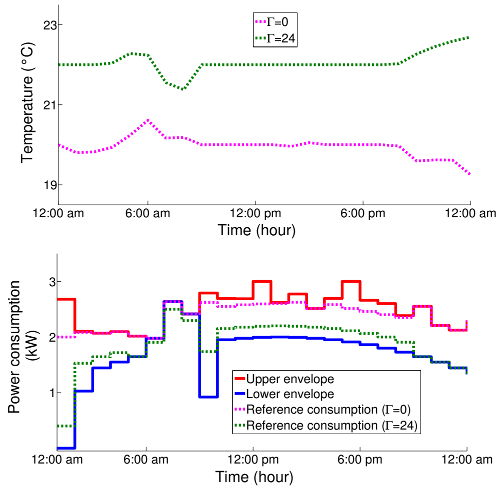

Fig. 4. Comparison of reference temperature and power consumption along with up and down reserve for $\Gamma = 0$ and $\Gamma = 24$, given $\delta = 0.1$.

are summarized in Table I. The results indicate the user can reduce the energy payment with the robust scheme if the worst case occurs for price uncertainty.

2) Case II: We compare the two solutions with respect to 200 electricity price realizations that were randomly generated based on the Γ coefficient. In Table I, the results of Case II show the average energy payment under different realizations of real time price. According to the results, the robust solution may lead to less energy payment for values of the Γ coefficient greater than 13. The comparison results for Case I and Case II are also represented in Figure 5. As shown in Figure 6, the standard deviation of the robust solution is less than the deterministic one and it generally decreases by Γ although the mean value of energy payment is increased.

To clarify the effect of comfort satisfaction weight factor on the reserve capacity, the problem is solved for different ranges of δ, which is shown in Figure 7. There is a turning point for each case, where the amount of up and down reserve are identical. After this point, the up reserve becomes larger than the down reserve. The turning point occurs at around $\Gamma = 12$ as the comfort satisfaction is not very important (e.g. given $\delta = 0$ in loose comfort satisfaction) and it takes place at larger

TABLE I
COMPARISON OF DAILY ENERGY PAYMENT (¢) VERSUS (Γ).

Γ	CaseI-deterministic	CaseI-robust	CaseII-deterministic	CaseII-robust
0	44.15	44.15	44.15	44.15
1	49.75	49.75	49.24	49.24
2	55.35	55.33	54.45	54.49
3	60.95	60.88	59.48	59.61
4	66.53	66.37	64.89	65.15
5	72.08	71.78	69.96	70.55
6	77.65	77.09	74.83	75.79
7	83.15	82.28	79.87	81.01
8	88.65	87.32	85.09	86.44
9	94.07	92.22	90.21	91.62
10	99.48	96.96	95.38	96.59
11	104.81	101.52	100.64	101.41
12	110.13	105.85	105.63	105.85
13	115.33	109.89	110.86	109.88
14	120.53	113.86	115.88	113.82
15	125.58	117.80	121.08	117.65
16	130.64	121.67	126.23	121.36
17	135.53	125.42	131.41	124.93
18	140.42	129.13	136.50	128.32
19	145.12	132.66	141.65	131.60
20	149.82	136.03	146.75	134.76
21	154.31	139.22	151.92	137.88
22	158.79	142.22	157.05	140.99
23	163.05	144.99	162.17	143.67
24	167.31	146.02	167.31	146.02

values of Γ as the comfort weight factor becomes of utmost importance to the user (e.g. given $\delta = 0.1$ in mild comfort satisfaction). This behavior is due to the lower sensitivity of the system to the price uncertainty for large values of δ. The possible up/down reserve and reference energy consumption will change with δ, as shown in Figure 8. In fact, with an increase in the value of δ, the sensitivity of the system to the price uncertainty and Γ is reduced. It also shows that the daily reference energy consumption is equal in both cases, $\Gamma = 0$ and $\Gamma = 24$, given $\delta = 10$, and the up/down flexibility of the system is almost negligible. Figure 9 depicts the total comfort violation across a day for a range of weight factors δ. The blue bar demonstrates the maximum total allowed deviation of indoor temperature from the desired level, in which the positive and negative deviations are calculated based on $\sum_{t \in T}(x_t^+ - \hat{x}_t)$ and $\sum_{t \in T}(\hat{x}_t - x_t^-)$, respectively. We see that the maximum comfort violation is a decreasing function of δ. The other bars, in pink, green, and red, compare the total deviation of reference temperature x_t from its expected level $\hat{x}_t$ across a day for different values of Γ as δ increases. For a better evaluation of the total comfort violation, both the positive and negative deviation sides are illustrated separately in Figure 9.

B. Long term analysis with different uncertainty sets

In this part, the same analysis was accomplished for a period of 7 days. According to the historical data analysis, the real time price may experience different hourly deviations from the day-ahead price. Larger deviations often take place in the afternoon, while during the morning, the real time price does not deviate noticeably from its predicted value. Therefore, for the simulation, the 24-hour time horizon across a day was divided into multiple parts with an appropriate price deviation p_t^+ for each part, which is captured from the

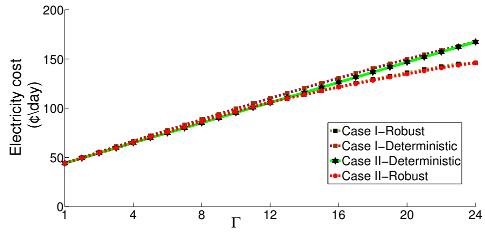

Fig. 5. Comparison of deterministic and robust solutions.

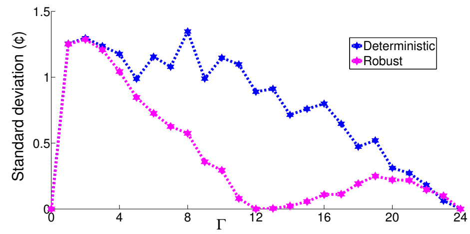

Fig. 6. Standard deviation of energy payment in Case II for different Γ.

historical data. The uncertainty set may be defined in many ways, such as using a priori assumptions, empirical statistics, linear regression, and machine learning [16]. In this part, we simply assume to create an uncertainty set using empirical statistics of the historical data of the electricity price. Hence, the problem was solved for $\Gamma = 4$, as the observation of the historical electricity prices indicates a significant deviation of price is more probable between 17:00 PM to 20:00 PM with approximately an average value of 4. During the other time slots, p_t^+ was assumed to be zero, as the average deviation of the price during those hours was very small in the past days. In fact, the problem becomes equivalent to the deterministic one, whenever the uncertainty set is small enough to be neglected. Figure 11 depicts the electricity price and reserve rewards for seven typical days in summer. The daily energy payment of the deterministic and robust schemes are compared for loose comfort satisfaction, in Figure 10. Based on this analysis, the robust solution may lead to a 5% saving in the energy payment compared with the deterministic approach.

V. CONCLUSIONS

Renewable energy sources are successfully integrated into the smart grid as one of the pillars of future energy systems. However, the major drawback of renewables is the intermittency and the uncertainty of their output, which can deteriorate the functionality of the grid. To this end, in this paper, demand-side flexibility has been studied to help mitigate the aforementioned flaws with a particular focus on electricity price uncertainty to incentivize more flexible consumption in building sectors. Knowing the rewards for each kW of reserve capacity provided and the day-ahead electricity price, the building system operator offers a day-ahead up/down reserve of the building HVAC system. According to the contract, the user will be charged based on a real time price. Thus, to minimize the user's total cost, a robust optimization model is proposed to determine the appropriate reserve subjects to the price uncertainty set. The robust optimization problem is a min-max problem, which was cast to a single-level minimization problem. To make the problem less conservative,

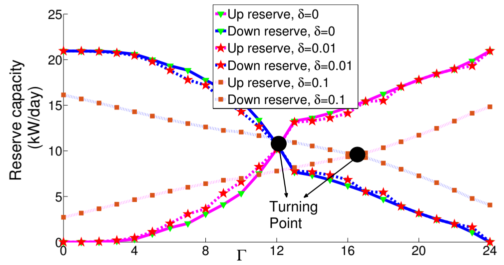

Fig. 7. Comparison of up and down reserve for different Γ.

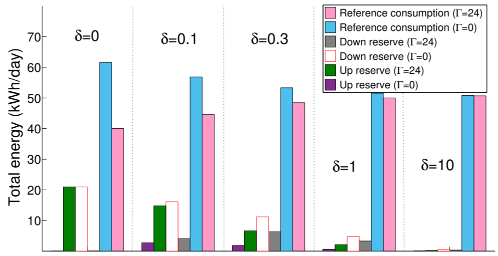

Fig. 8. Comparison of daily up/down reserve and reference energy consumption for different δ.

we assumed to estimate the hourly uncertainty set based on the historical data so as to take only those time slots with higher probability of uncertainty occurrence into account. The impact of comfort weight factor on the decision making was elaborated by comparing the up/down reserve and total comfort violations. As evidenced by the simulation results, the robust optimization approach may reduce the electricity payment especially when subject to a drastic price uncertainty.

VI. Appendix

In this part, we show how problem (5) is simplified into problem (6a), using the Lagrangian duality. In problem (5), the inner maximization is a Linear Program (LP). To form the Lagrangian, we introduce three multipliers ζ_t, μ_t, and β, for the inequality constraint (4b) and obtain the Lagrangian function as

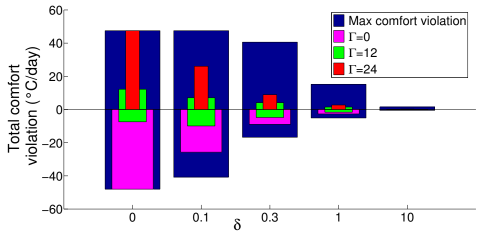

Fig. 9. Comparison of total comfort violation for different δ.

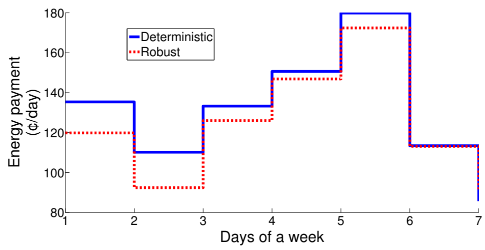

Fig. 10. Energy payment comparison for the deterministic and robust solutions for a week in summer, given $\delta = 0$.

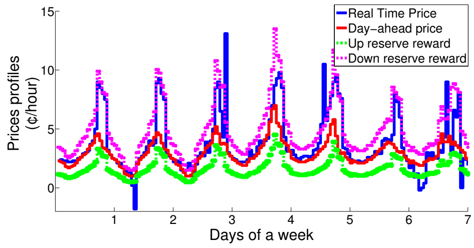

Fig. 11. Day-ahead and real time price, and reserve rewards in a week.

$$L(\epsilon_t, \zeta_t, \mu_t, \beta) = \sum_{t \in T} \{p_t^+ \epsilon_t u_t + \zeta_t (1 - \epsilon_t) + \mu_t \epsilon_t\} + \beta(\Gamma - \sum_{t \in T} \epsilon_t)$$
$$= \beta\Gamma + \sum_{t \in T} \zeta_t + \sum_{t \in T} \epsilon_t (p_t^+ u_t - \zeta_t + \mu_t - \beta)$$

L is a linear function of ϵ_t and it is bounded if the term multiplying ϵ_t is identically zero. Thus, $p_t^+ u_t - \zeta_t + \mu_t - \beta = 0$ and the dual problem is the LP

$$\min_{\beta, \zeta_t} \quad \beta\Gamma + \sum_{t \in T} \zeta_t$$
$$\textbf{subject to} \quad \mu_t = \beta + \zeta_t - p_t^+ u_t \geq 0 \quad , \quad \zeta_t, \beta \geq 0$$

Combining the above result with the outer minimization in problem (5) results in a single-level minimization problem formulated as (6a).

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